Optimal Resource Allocation in OFDMA Downlink Systems with Imperfect CSI

Rohit Aggarwal, Mohamad Assaad, C. Emre Koksal, and Phil Schniter

SPAWC 2011

Problem:

In an OFDMA downlink, we want to

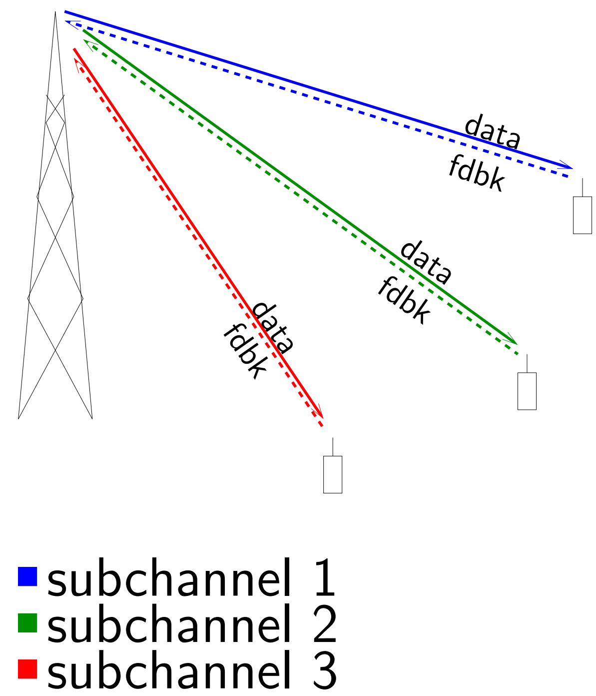
- schedule subchannels to users,
- allocate power among users, and
- assign coding schemes to users,

in order to

- maximize a goodput-based utility

subject to

- a total power constraint, and
- uncertainty in the subchannel gains.



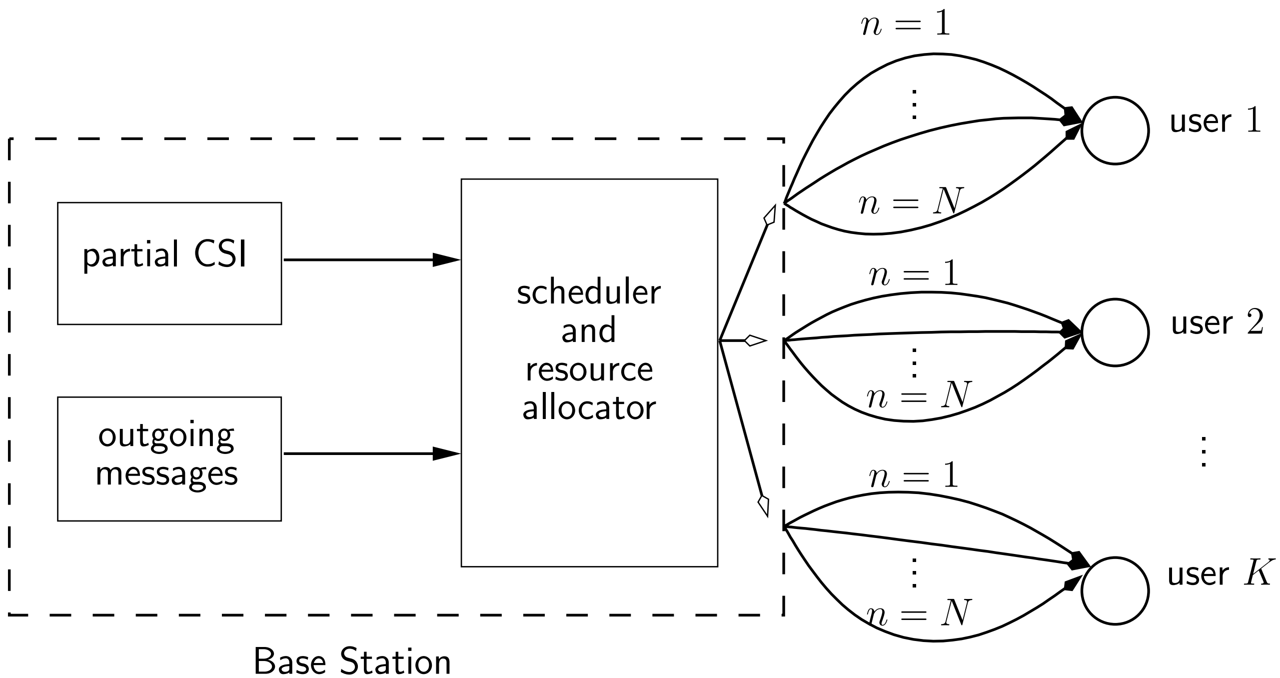
Contributions:

1. (Near) optimal resource allocation algorithms under arbitrary CSI distributions for two scenarios: with/without subcarrier time-sharing.
2. Faster than state-of-the-art algorithms from

   [1] Huang, Subramanian, Agrawal, Berry, "Downlink scheduling and resource allocation for OFDM systems," IEEE TWC Jan. 2009.

   [2] Wong and Evans "Optimal resource allocation in the OFDMA downlink with imperfect channel knowledge," IEEE TCOM Jan. 2009.
3. Tight bounds on the performance of our proposed algorithms.
4. Our general goodput-based utility framework encompasses, e.g., optimization with regard to
   - capacity,
   - throughput of a practical coding scheme, or
   - differentiated pricing formulations across applications or users.

System Model:



N : # of OFDM subchannels

K : # of users

M : # of coding schemes

Our Approach:

Goodput-based utility:

- Goodput $\boxed{g = (1-\epsilon) r_m}$ is the number of bits-per-channel-use communicated *without error*.
- Error rate $\boxed{\epsilon = a_m e^{-b_m p \gamma}}$ for power p, SNR γ, and constants a_m, b_m that vary with coding scheme m.
- Utility $U_{k,m}(g)$ is any concave and strictly-increasing function of g. Can be user (k) and coding-scheme (m) dependent.

Imperfect CSI:

- We assume an arbitrary *distribution* on the SNR γ.

Scheduling and resource allocation:

- We maximize *expected* sum-utility subject to a total-power constraint.

Problem Formulation:

$$\max_{\substack{\{p_{n,k,m}\geq 0\}\\ \{I_{n,k,m}\in\mathcal{I}\}}} \mathrm{E}\left\{\sum_{n=1}^{N}\sum_{k=1}^{K}\sum_{m=1}^{M} I_{n,k,m}U_{k,m}\big((1-a_m e^{-b_m p_{n,k,m}\gamma_{n,k}})r_m\big)\right\}$$

$$\text{s.t.} \quad \sum_{k,m} I_{n,k,m} \leq 1 \ \forall n \quad \text{and} \quad \sum_{n,k,m} I_{n,k,m}\, p_{n,k,m} \leq P_{\mathsf{con}}$$

where

$$\begin{aligned} I_{n,k,m} &= \text{time-share of } n^{th} \text{ subchannel by user/code } (k,m), \\ p_{n,k,m} &= \text{power allocated to user/code } (k,m) \text{ on } n^{th} \text{ subchannel.} \\ \gamma_{n,k} &= \text{SNR of user } k \text{ on } n^{th} \text{ subchannel.} \\ r_m &= \text{rate of } m^{th} \text{ coding scheme.} \end{aligned}$$

We consider two problem formulations:

Continuous : subchannel time-sharing is allowed: $I_{n,k,m} \in [0,1] \triangleq \mathcal{I}$.

Discrete : subchannel time-sharing is not allowed: $I_{n,k,m} \in \{0,1\} \triangleq \mathcal{I}$.

Remarks:

1. As stated, the optimization problem is not convex.
2. In the continuous case, the problem can be convexified by substituting $p_{n,k,m} = \frac{x_{n,k,m}}{I_{n,k,m}}$ and optimizing over $\{x_{n,k,m}\}$ and $\{I_{n,k,m}\}$.
3. In the discrete case, we have a mixed-integer optimization problem. Such problems are (in general) NP-hard.
4. If the schedule $\boldsymbol{I} = \{I_{n,k,m}\}$ is fixed, then resource (i.e., power) allocation is a convex optimization problem.

Continuous Scheduling and Resource Allocation:

$$\max_{\substack{\{p_{n,k,m}\geq 0\}\\ \{I_{n,k,m}\in[0,1]\}}} \mathrm{E}\left\{\sum_{n=1}^{N}\sum_{k=1}^{K}\sum_{m=1}^{M} I_{n,k,m}F_{n,k,m}(I_{n,k,m},x_{n,k,m})\right\}$$

$$\text{s.t.}\quad \sum_{k,m} I_{n,k,m} \leq 1\ \forall n \quad\text{and}\quad \sum_{n,k,m} x_{n,k,m} \leq P_{\mathsf{con}}$$

where

$$F_{n,k,m}(I_{n,k,m},x_{n,k,m}) = \begin{cases} -\mathrm{E}\left\{U_{k,m}\big((1-a_m e^{-b_m \frac{x_{n,k,m}}{I_{n,k,m}}\gamma_{n,k}})r_m\big)\right\} & \text{if } I_{n,k,m}\neq 0\\ 0 & \text{otherwise.}\end{cases}$$

Remarks:

1. This is a convex optimization problem with $N+1$ constraints.
2. The KKT conditions show that the dual variables corresponding to the subchannel-resource constraint are redundant.

Dual Formulation of Continuous Problem:

The Lagrangian is

$$L(\mu, \boldsymbol{I}, \boldsymbol{x}) := \sum_{n,k,m} I_{n,k,m} \, F_{n,k,m}(I_{n,k,m}, x_{n,k,m}) + \Big(\sum_{n,k,m} x_{n,k,m} - P_{\textsf{con}} \Big) \mu$$

where

$$F_{n,k,m}(I_{n,k,m}, x_{n,k,m}) = \begin{cases} -\operatorname{E}\left\{ U_{k,m}\big((1 - a_m e^{-b_m \frac{x_{n,k,m}}{I_{n,k,m}} \gamma_{n,k}}) r_m\big) \right\} & \text{if } I_{n,k,m} \neq 0 \\ 0 & \text{otherwise.} \end{cases}$$

The corresponding dual problem is:

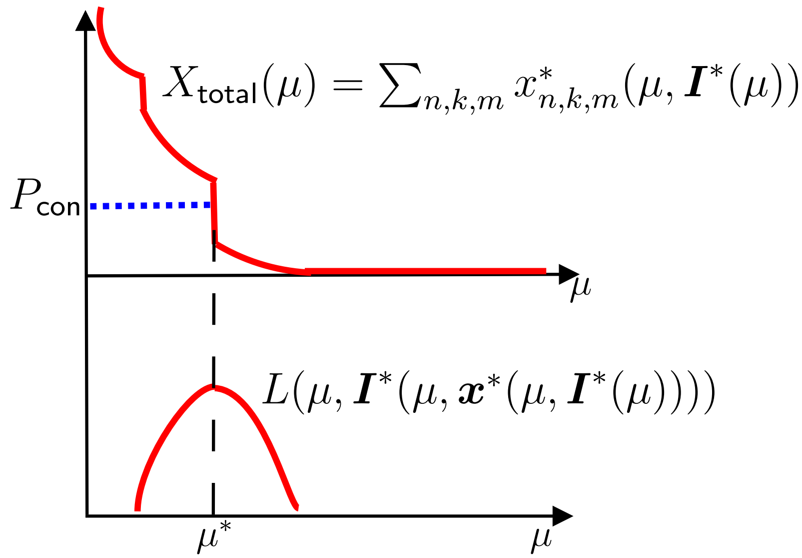
$$\max_{\mu \geq 0} \min_{\boldsymbol{I} \in \boldsymbol{\mathcal{I}}} \min_{\boldsymbol{x} \succeq 0} L(\mu, \boldsymbol{I}, \boldsymbol{x})$$

One then finds...

- $\boldsymbol{x}^*(\mu, \boldsymbol{I})$: optimal powers for a given $(\mu, \boldsymbol{I})$,
- $\boldsymbol{I}^*(\mu)$: optimal schedule for a given μ,
- μ^* : optimal Lagrange multiplier μ.

Important Observations:

Lemma 1 *The optimal total-power allocation is a monotonically decreasing function of* μ.



Lemma 2 μ^* *lives in the interval* $[\mu_{\mathit{min}}, \mu_{\mathit{max}}]$, *where*

$$\mu_{\mathit{min}} = \min_{n,k,m} a_m b_m r_m \,\mathrm{E}\left\{U'_{k,m}\big((1 - a_m e^{-b_m P_{\mathit{con}} \gamma_{n,k}}) r_m\big) \gamma_{n,k} e^{-b_m P_{\mathit{con}} \gamma_{n,k}}\right\}$$

$$\mu_{\mathit{max}} = \max_{n,k,m} a_m b_m r_m U'_{k,m}\big((1 - a_m) r_m\big) \,\mathrm{E}\{\gamma_{n,k}\}$$

Bisection-based Algorithm for the Continuous Problem:

Initialize with $\mu_{\text{upper}} = \mu_{\text{max}}$ and $\mu_{\text{lower}} = \mu_{\text{min}}$.

1. Set $\mu \leftarrow \frac{\mu_{\text{upper}}+\mu_{\text{lower}}}{2}$.
2. Calculate $x^*_{n,k,m}(\mu, \boldsymbol{I}^*(\mu))$ and $I^*_{n,k,m}(\mu)$ for all (n,k,m).
3. Calculate $X_{\text{total}}(\mu) = \sum_{n,k,m} x^*_{n,k,m}(\mu, \boldsymbol{I}^*(\mu))$.
4. If $X_{\text{total}}(\mu) < P_{\text{con}}$, set $\mu_{\text{upper}} \leftarrow \mu$, otherwise set $\mu_{\text{lower}} \leftarrow \mu$.

Repeat Steps 1–4 until $\mu_{\text{upper}} - \mu_{\text{lower}} < \kappa$, where κ is a stopping parameter.

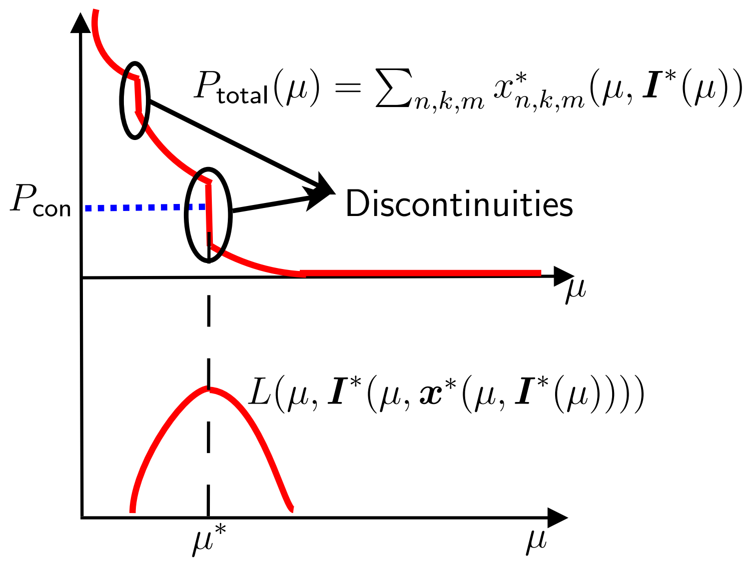
Performance Guarantee:

$$U^*_{\text{cont}} - \hat{U}_{\text{cont}}(\kappa) \leq P_{\text{con}}\kappa,$$

where

- U^*_{cont} is the optimal sum-utility, and
- $\hat{U}_{\text{cont}}(\kappa)$ is the sum-utility achieved by the above algorithm for a given κ.

Additional Important Observations:



Lemma 3

a. *At a discontinuity, there exists some subchannel n at which the optimal schedule time-shares several user/code combinations (k, m).*

b. *Otherwise, at most one user/code combination (k, m) is scheduled for every subchannel n, and the corresponding allocation solves the discrete problem for the total power constraint $P_{\text{con}} = P_{\text{total}}(\mu)$.*

c. *The number of discontinuities at most countable.*

Bisection-based Algorithm for the Discrete Problem:

1. Run the proposed continuous algorithm for stopping criterion κ, yielding $\mu^* \in [\mu_{\text{lower}}, \mu_{\text{upper}}]$ with $\mu_{\text{upper}} - \mu_{\text{lower}} < \kappa$.
2. Solve the power allocation problem for each of the two schedules $\{\boldsymbol{I}^*(\mu_{\text{lower}}), \boldsymbol{I}^*(\mu_{\text{upper}})\}$ and choose the utility-maximizing one.

<u>Performance Guarantee:</u>

$$U^*_{\text{discrete}} - \lim_{\kappa \to 0} \hat{U}_{\text{discrete}}(\kappa) \;\leq\; (\mu^* - \mu_{\text{min}})\big(P_{\text{con}} - X^{\text{min}}_{\text{total}}(\mu^*)\big),$$

where

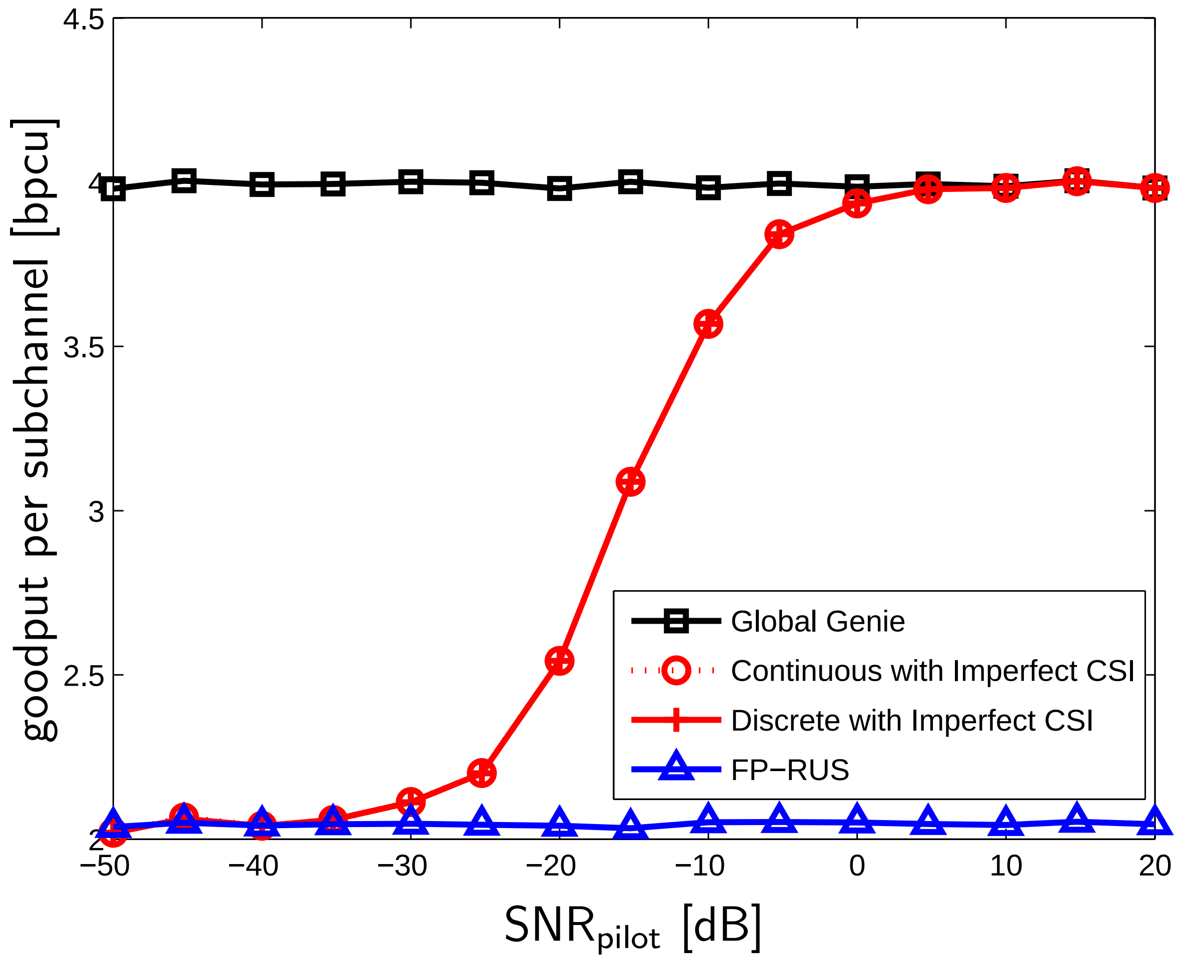
- U^*_{discrete} is the optimal utility for the discrete allocation problem,
- $\hat{U}_{\text{discrete}}(\kappa)$ is the utility achieved by the proposed algorithm, and
- $X^{\text{min}}_{\text{total}}(\mu^*)$ is the min value of $X_{\text{total}}(\mu)$ at μ in case a discontinuity exists.

Numerical Experiments:

- $N = 64$ subchannels
- $K = 16$ users.
- Uncoded 2^{m+1}-QAM with $m \in \{1, \ldots, 15\}$: $M = 15$ and $r_m = m + 1$ and error-rate params $a_m = 1$, $b_m = 1.5/((m+1)^2 - 1)$.
- Rayleigh-fading subchannels $h_{n,k} \sim \mathcal{CN}(0, 1)$ with $\gamma_{n,k} = |h_{n,k}|^2$.
- Uncertain CSI: channel inference from pilots yields a non-central chi-squared posterior distribution on $\gamma_{n,k}$.
- Nominally, $\text{SNR} = 10$ dB and $\text{SNR}_{\text{pilot}} = -10$ dB.
- Algorithmic stopping criterion: $\kappa = 0.3/P_{\text{con}}$.
- Reference schemes for comparison:
  - *Global Genie*: has perfect knowledge of $\gamma_{n,k}$ realizations.
  - *Fixed-Power Random-User-Scheduling* (FP-RUS): uses prior channel statistics.

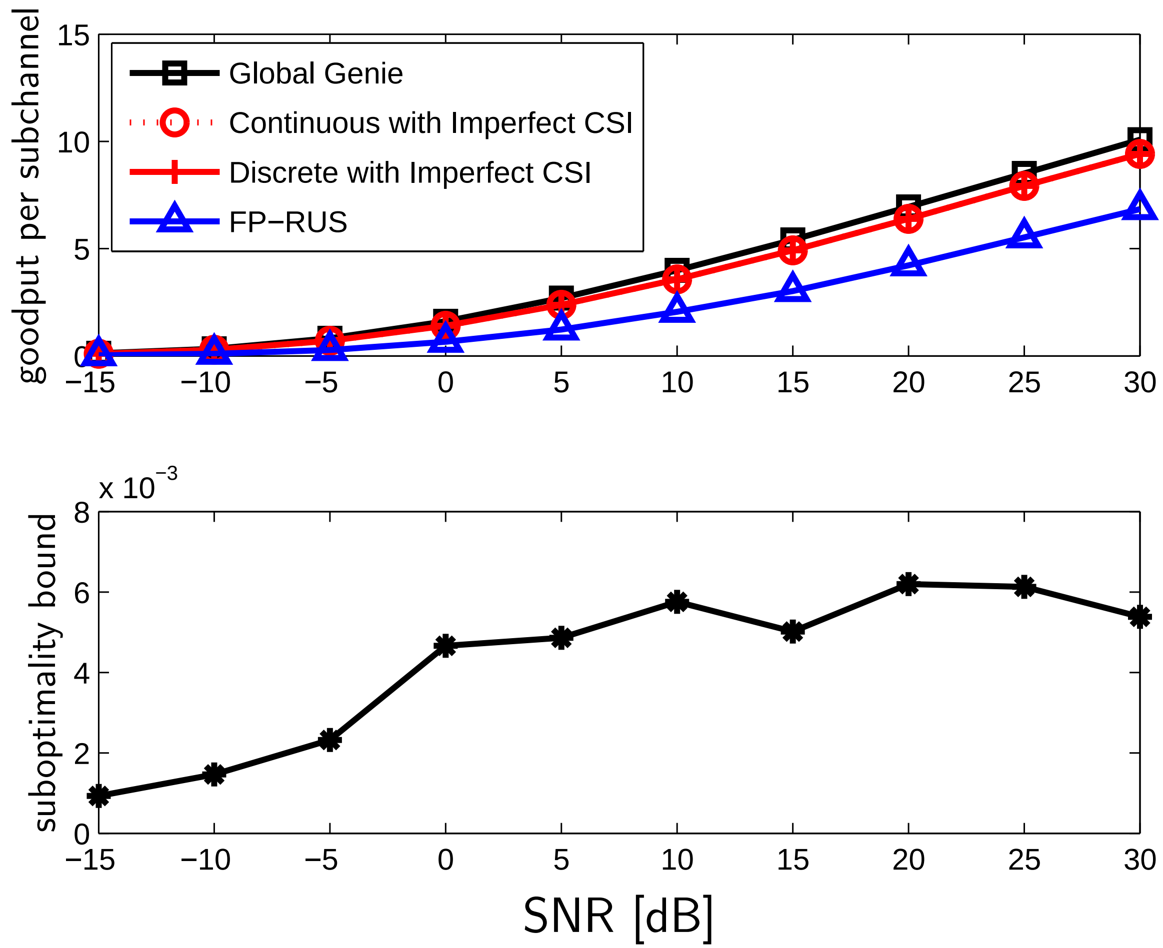
Sum-goodput versus pilot-SNR:

Sum-goodput maximization: $U_{k,m}(g) = g \quad \forall k, m.$



Sum-goodput (and suboptimality bound) versus SNR:

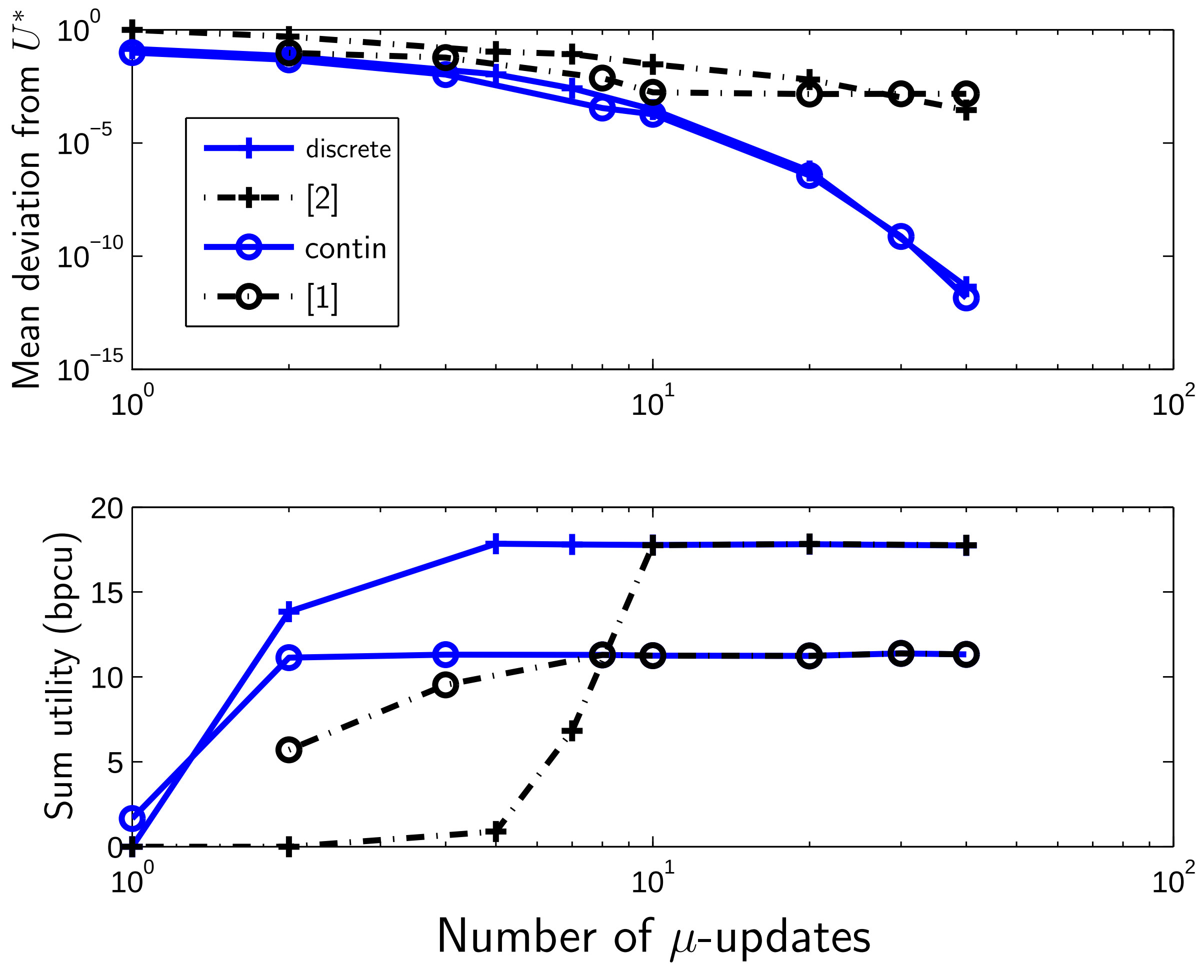
Sum-goodput maximization: $U_{k,m}(g) = g \quad \forall k, m.$



Performance versus [1] and [2]:

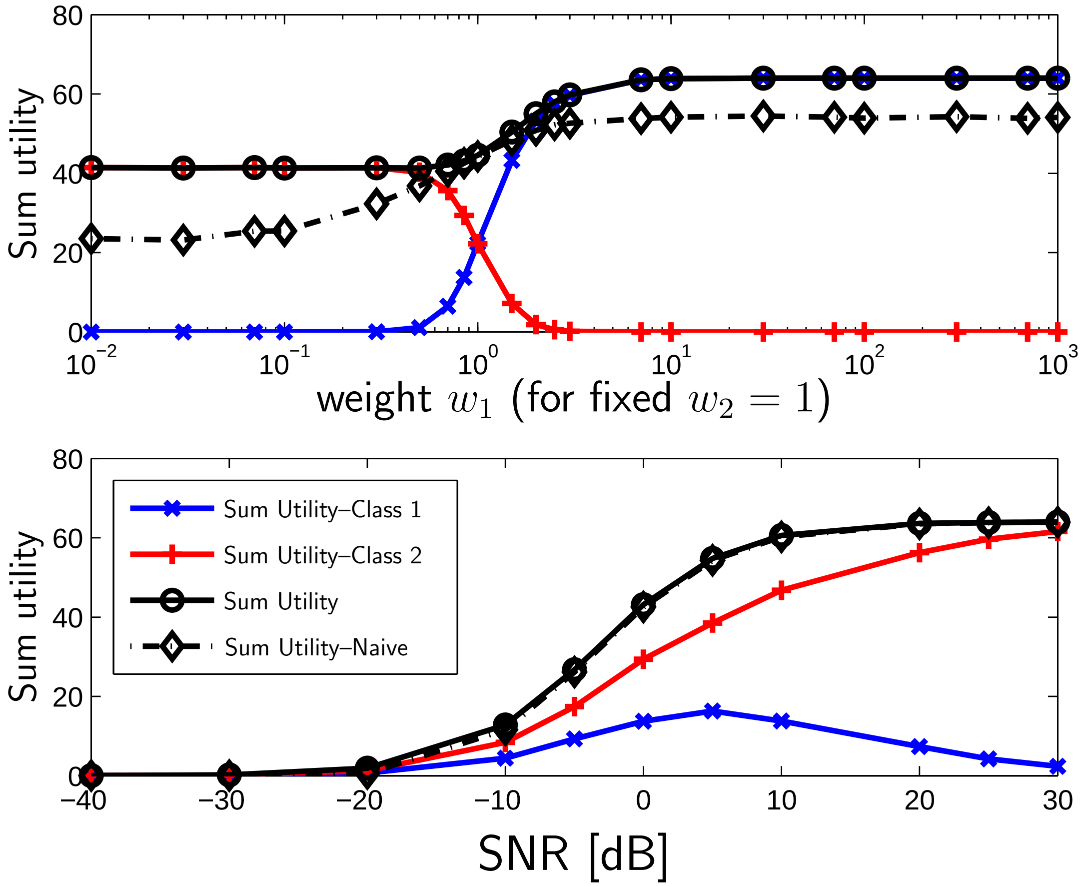
Continuous case: maximization of upper bound on capacity [1].

Discrete case: maximization of sum-capacity [2].



Example of a pricing-based utility:

$$U_{k,m}(g) = \begin{cases} 1 - e^{-w_1 g} & k \in \mathcal{K}_1 \\ 1 - e^{-w_2 g} & k \in \mathcal{K}_2 \end{cases}; \text{ users in } \mathcal{K}_1 \text{ prioritized when } w_1 > w_2.$$



Conclusions:

- We considered OFDMA scheduling-and-resource-allocation to maximize goodput-based utilities under generic CSI distributions.
- Our goodput-based utility framework can handle, e.g., optimization w.r.t sum-capacity, sum-throughput for practical coding schemes, or differentiated pricing-models.
- Two flavors of the problem were considered: 1) “continuous” sharing of subchannels, and 2) “discrete” assignment (no sharing).
- Bisection-based algorithms were given for both problems that are significantly faster than the state-of-the-art.
- Tight bounds on suboptimality were provided.
- Numerical experiments confirmed the excellent behavior of the proposed algorithms.