Efficient Sequence Detection of Multi-Carrier Transmissions over Doubly Dispersive Channels

Phil Schniter

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Multicarrier System Model:

Modulation:

$$s(t) = \sum_{n=-\infty}^{\infty} \sum_{k=0}^{N-1} [\boldsymbol{s}_n]_k \, a(t - nT_s) e^{j2\pi k F_s (t - nT_s)}$$

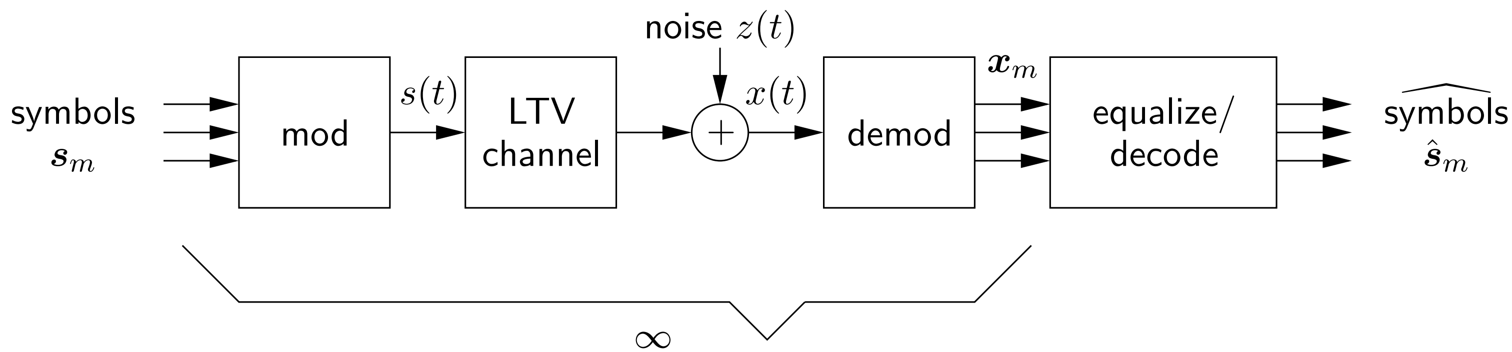
Doubly dispersive channel:

$$x(t) = \int_0^{T_h} h(t,\tau) s(t-\tau) d\tau + z(t)$$

Demodulation:

$$[\boldsymbol{x}_m]_k = \int_{-\infty}^{\infty} x(t) \, b^*(t - mT_s) e^{-j2\pi k F_s (t - mT_s)}$$

Discrete-time Vector Representation:



$$\boldsymbol{x}_m = \sum_{n=-\infty}^{\infty} \boldsymbol{H}_{m,n} \boldsymbol{s}_{m-n} + \boldsymbol{z}_m$$

"ISI+ICI channel"

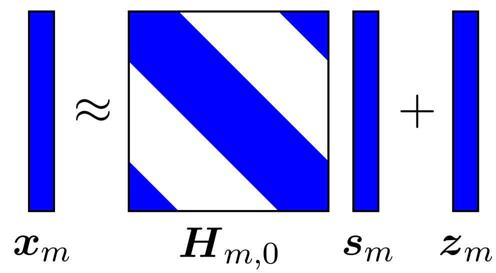
$\boldsymbol{s}_m \in \mathbb{C}^N$	multi-carrier symbol vector
$\boldsymbol{H}_{m,n} \in \mathbb{C}^{N \times N}$	sub-carrier coupling matrix at time-m and lag-n
$\boldsymbol{x}_m \in \mathbb{C}^N$	multi-carrier observation vector
$\boldsymbol{z}_m \in \mathbb{C}^N$	noise vector

Quasi-Banded Model:

With properly chosen pulse shapes $a(t)$ and $b(t)$, and with a smoothly varying channel, we can make the approximation

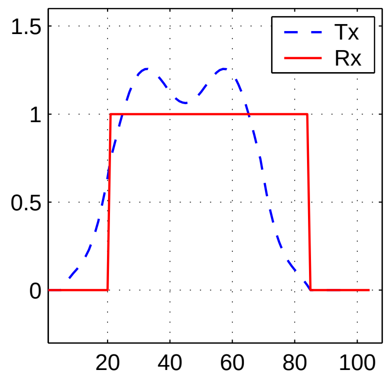
$$\begin{aligned} \boldsymbol{x}_m &= \sum_{n=-\infty}^{\infty} \boldsymbol{H}_{m,n}\boldsymbol{s}_{m-n} + \boldsymbol{z}_m \\ &\approx \boldsymbol{H}_{m,0}\boldsymbol{s}_m + \boldsymbol{z}_m \end{aligned}$$

where $\boldsymbol{H}_{m,0}$ is quasi-banded with $2D+1$ active diagonals:

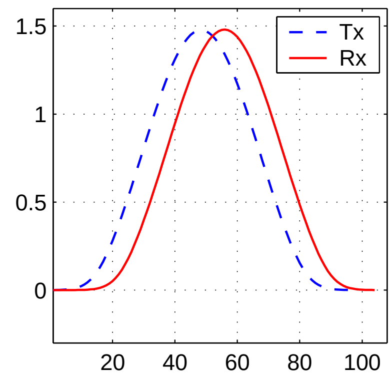


In other words, ISI becomes negligible and ICI is effectively limited to a radius of D subcarriers.

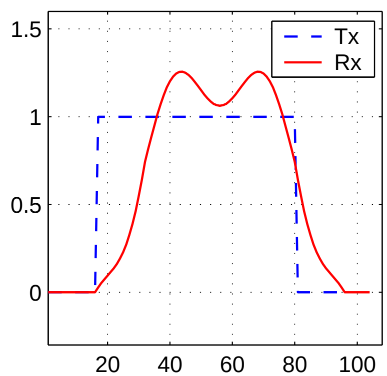
In fact, with knowledge of the channel statistics, the pulses $\{a(t), b(t)\}$ can be designed to make the approximation accurate (without compromising spectral efficiency). Example max-SINR pulse designs:



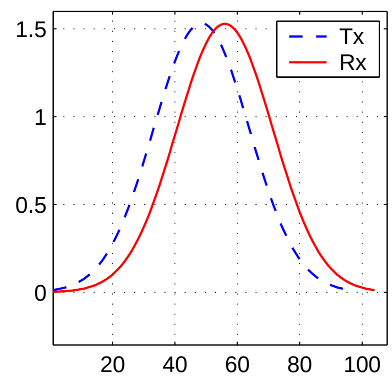
Transmitter optimized



Jointly optimized

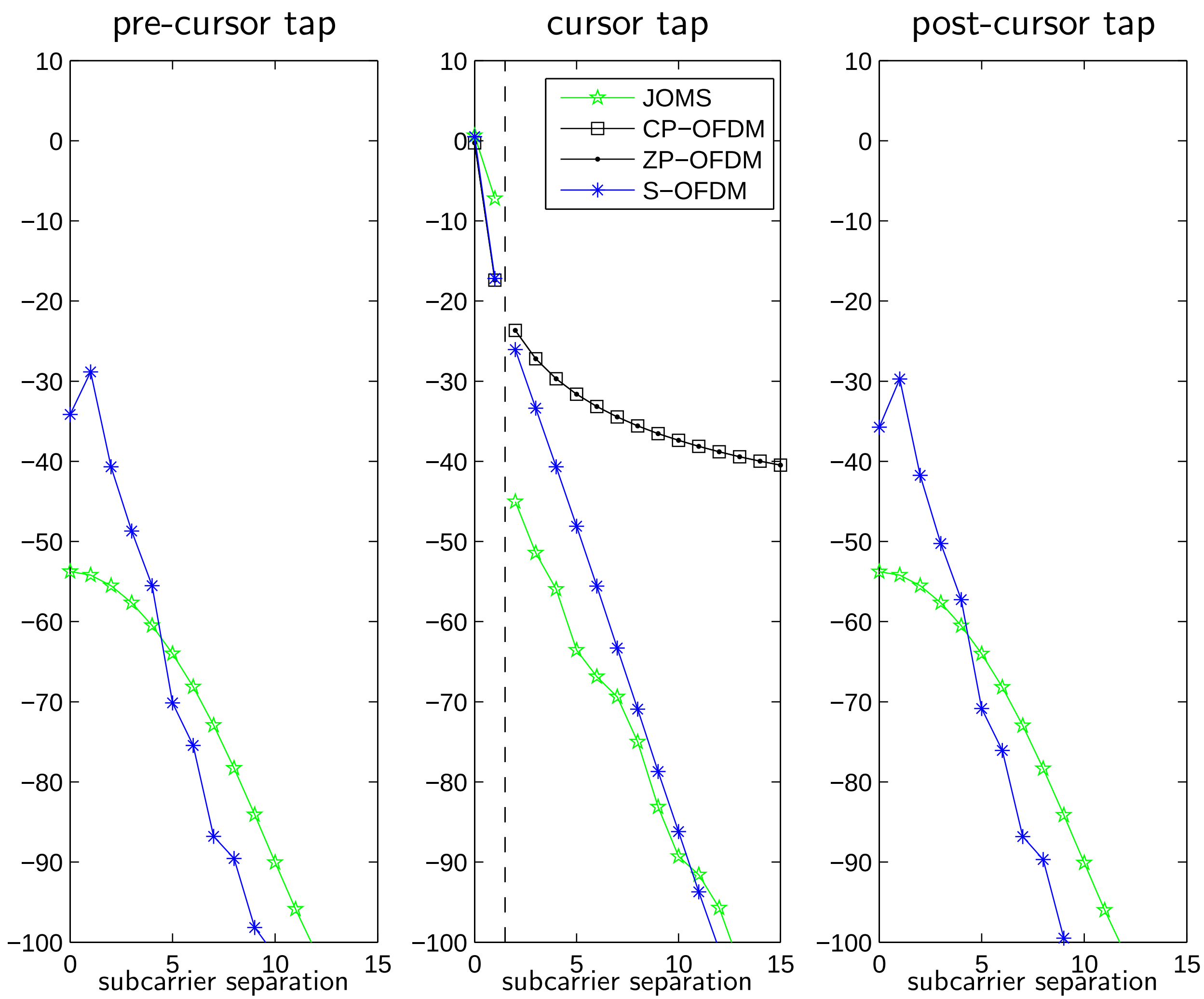


Receiver optimized



Gaussian

ICI Energy vs. Subcarrier Separation:



Efficient Symbol Detection:

Prior art:

1. Linear (e.g., MMSE, ZF) [Rugini/Banelli/Leus SPL 05] $\mathcal{O}(D^2N)$
2. DFE [Rugini/Banelli/Leus SPAWC 05] $\mathcal{O}(D^2N)$
3. Iterative [Schniter TSP 04] $\mathcal{O}(D^2N)$
4. ML (e.g., Viterbi) [Matheus/Kammeyer GLOBE 97] $\mathcal{O}(M^DDN)$

where M is the constellation size.

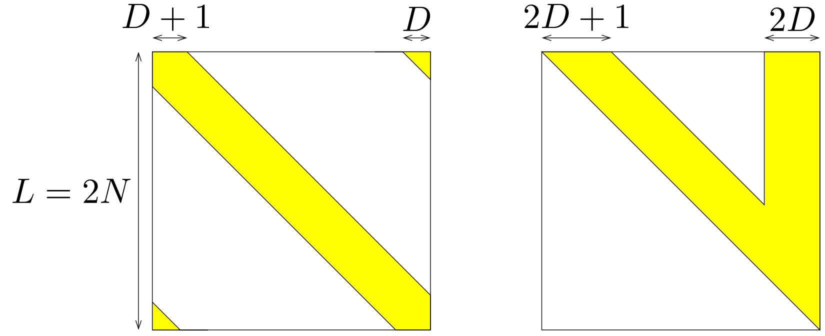
Can we get ML-like performance with DFE-like complexity?

Yes, via sequential decoding (i.e., tree search or closest point lattice search)!

Sequential Decoding (SqD):

Two-step procedure:

1. Pre-processing (to expose tree structure),



2. Efficient (possibly sub-optimal) tree search.

Both steps should leverage quasi-banded structure of ICI matrix for complexity reduction.

SqD Pre-Processing:

1. QR (traditional method): For $\boldsymbol{H} = \boldsymbol{Q}\boldsymbol{R}$ with unitary $\boldsymbol{Q}$ and upper triangular $\boldsymbol{R}$,

$$\hat{\boldsymbol{s}}_{\mathsf{ML}} = \arg\min_{\boldsymbol{s}\in\mathcal{S}^N} \|\boldsymbol{x} - \boldsymbol{H}\boldsymbol{s}\|^2 = \arg\min_{\boldsymbol{s}\in\mathcal{S}^N} \|\underbrace{\boldsymbol{Q}^H\boldsymbol{x}}_{\boldsymbol{x}'} - \boldsymbol{R}\boldsymbol{s}\|^2$$

   Problem: $\boldsymbol{R}$ may be ill-conditioned, in which case sub-optimal tree search tends to be costly. [Murugan/El-Gamal TIT 06]

2. MMSE-GDFE [Damen/El-Gamal CISS 04]: For $\begin{pmatrix}\boldsymbol{H}\\ \gamma^{-1/2}\boldsymbol{I}_N\end{pmatrix} = \begin{pmatrix}\boldsymbol{Q}_1\\ \boldsymbol{Q}_2\end{pmatrix}\boldsymbol{R}$ with unitary $\begin{pmatrix}\boldsymbol{Q}_1\\ \boldsymbol{Q}_2\end{pmatrix}$ and upper triangular $\boldsymbol{R}$,

$$\hat{\boldsymbol{s}}_{\mathsf{PP}} = \arg\min_{\boldsymbol{s}\in\mathcal{S}^N} \|\underbrace{\boldsymbol{Q}_1^H\boldsymbol{x}}_{\boldsymbol{\rho}} - \boldsymbol{R}\boldsymbol{s}\|^2 \neq \hat{\boldsymbol{s}}_{\mathsf{ML}}$$

   Note: $\hat{\boldsymbol{s}}_{\mathsf{PP}} = \hat{\boldsymbol{s}}_{\mathsf{ML}}$ under QPSK & BPSK [Hwang/Schniter ALL 05].

Fast MMSE-GDFE Pre-Processing:

Steps:

1. Compute $\begin{pmatrix} \boldsymbol{H} \\ \gamma^{-1/2}\boldsymbol{I}_N \end{pmatrix}^H \begin{pmatrix} \boldsymbol{H} \\ \gamma^{-1/2}\boldsymbol{I}_N \end{pmatrix}$ $\mathcal{O}(D^2N)$
2. Cholesky factorize $\begin{pmatrix} \boldsymbol{H} \\ \gamma^{-1/2}\boldsymbol{I}_N \end{pmatrix}^H \begin{pmatrix} \boldsymbol{H} \\ \gamma^{-1/2}\boldsymbol{I}_N \end{pmatrix} = \boldsymbol{R}^H\boldsymbol{R}$ $\mathcal{O}(D^2N)$
3. Compute $\boldsymbol{b} := \boldsymbol{H}^H\boldsymbol{x} = \boldsymbol{R}^H\boldsymbol{\rho}$ $\mathcal{O}(DN)$
4. Forward substitute to get $\boldsymbol{\rho}$ from $\boldsymbol{b}$ $\mathcal{O}(DN)$

Note: Similar to fast MMSE-DFE from [Rugini/Banelli/Leus SPAWC 05] but designed for quasi-banded (rather than banded) matrices.

Additional Pre-Processing?

- Additional pre-processing stages, such as lattice reduction and column ordering (e.g., V-BLAST), are common in SqD. However, most of them destroy the quasi-banded structure we need for fast MMSE-GDFE, and so are not appropriate in our application.
- A simple circular shift in the column order is admissible. We find that rotating the strongest column into the rightmost position in $\boldsymbol{R}$ yields a small improvement in the performance/complexity of the subsequent tree search.

Tree Search:

Now we focus on solving

$$\hat{\boldsymbol{s}}_{\mathsf{PP}} \;=\; \arg\min_{\boldsymbol{s}\in\mathcal{S}^N} \|\boldsymbol{\rho} - \boldsymbol{R}\boldsymbol{s}\|^2 \quad \text{with "V-shaped" } \boldsymbol{R},$$

i.e., efficiently searching a tree with M^N leaf nodes.

Options:

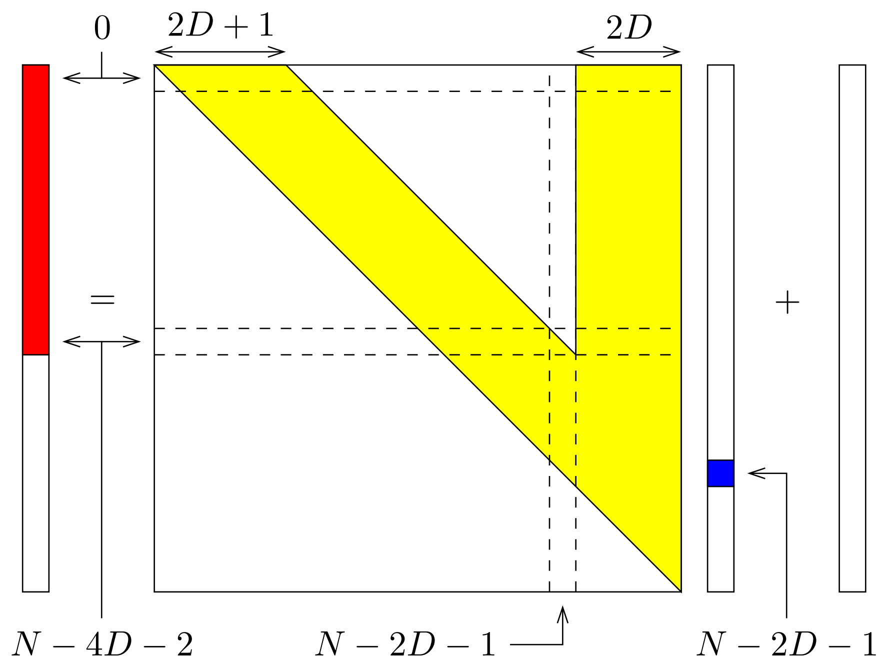
1. Depth-first search (e.g., Schnorr-Euchner sphere decoder)
2. Best-first search (e.g., Fano alg, stack alg)
3. Breadth-first search (e.g., M-alg, T-alg, Pohst sphere decoder)

Depth-first Search:

- Proceed down tree by following min-cost branch at each level. Keep first full path (i.e., DFE estimate) as a reference. Then, back up one level at a time and re-examine any discarded branches which have a chance at beating the reference. Reset reference if a better one is found, and repeat.
- Very efficient at high SNR, because DFE estimate is nearly ML and few paths need to be re-examined. At low SNR, many paths must be re-examined, leading to a complexity explosion.
- Additional problem with V-shaped $\boldsymbol{R}$: Symbol errors are not always visible in down-stream observations, meaning that back-tracking will need to go very deep to uncover errors.

Error masking in V-shaped $\boldsymbol{R}$:

Recall $\boldsymbol{\rho} = \boldsymbol{R}\boldsymbol{s} + \boldsymbol{n}$:



The symbol s_{N-2D-1} does not affect $\{\rho_0, \ldots, \rho_{N-4D-2}\}$!

Best-First Search:

- Maintain a sorted list of best partial paths (of possibly different lengths). At each iteration, replace best partial path with it's children and re-sort list. Terminate with best partial path is a full path.
- The Fano alg adds a user-selected bias towards longer paths, facilitating a performance/complexity trade-off. With a large enough bias, Fano becomes DFE.
- Fano alg known for excellent performance with fully populated upper triangular $\boldsymbol{R}$, but V-shaped $\boldsymbol{R}$ leads to inefficient search, due in part to the fact that the Fano bias rewards the extension of paths with early errors, e.g., errors in $\hat{s}_{N-2D-1}$.

Breadth-First Search:

- Proceed down the tree level-by-level, extending only the best partial paths at each level. Terminate when the last level is reached.
- The complexity of breath-first search is relatively insensitive to SNR and to the structure of $\boldsymbol{R}$.
- The M-alg investigates a fixed number of branches per level. This involves a compromise, however, since there is typically no single number that works well in all situations.
- The T-alg investigates the paths whose metrics are within some threshold T of the best path's metric at the current level. Several methods to choose T have been proposed, e.g., experimentally or based on SNR.

Channel-Adaptive T-Algorithm:

- We propose a new variant of the T-alg, where T_i, the threshold at the i^{th} level, is adjusted based on the channel realization and noise variance.

- The main idea is to discard the true path with probability at most ϵ_o when the true path is not the best partial path. In other words, T_i is chosen such that

$$\Pr\{\mathcal{M}(\boldsymbol{s}_{\mathsf{T}}^{(i)}) > \mathcal{M}(\boldsymbol{s}_{\star}^{(i)}) + T_i \;\big|\; \mathcal{M}(\boldsymbol{s}_{\mathsf{T}}^{(i)}) > \mathcal{M}(\boldsymbol{s}_{\star}^{(i)})\} < \epsilon_o$$

  Note: Simply setting T_i so that the true path is discarded with probability at most ϵ_o would allow too high a search complexity with difficult channels.

- The key to efficient search is *to know when to give up*; difficult channels are not worth expensive searches!

Channel-Adaptive T-Algorithm:

- We assume that the event $\mathcal{M}(\boldsymbol{s}_{\mathsf{T}}^{(i)}) > \mathcal{M}(\boldsymbol{s}_{\star}^{(i)})$ is dominated by the case that $\boldsymbol{s}_{\mathsf{T}}^{(i)}$ and $\boldsymbol{s}_{\star}^{(i)}$ differ in a single element at the weakest column of $\boldsymbol{R}^{(i)}$. We also assume that $\boldsymbol{\rho}^{(i)} - \boldsymbol{R}^{(i)}\boldsymbol{s}_{\mathsf{T}}^{(i)}$ is Gaussian. Under these assumptions, the threshold

$$T_i = 2\sigma_z \|\boldsymbol{r}_{\mathsf{weak}}^{(i)}\| \mathcal{Q}^{-1}\left(\epsilon_o \mathcal{Q}\left(\frac{\|\boldsymbol{r}_{\mathsf{weak}}^{(i)}\|}{2\sigma_z}\right)\right) - \|\boldsymbol{r}_{\mathsf{weak}}^{(i)}\|^2$$

ensures that

$$\Pr\{\mathcal{M}(\boldsymbol{s}_{\mathsf{T}}^{(i)}) > \mathcal{M}(\boldsymbol{s}_{\star}^{(i)}) + T_i \mid \mathcal{M}(\boldsymbol{s}_{\mathsf{T}}^{(i)}) > \mathcal{M}(\boldsymbol{s}_{\star}^{(i)})\} = \epsilon_o.$$

Numerical Experiments:

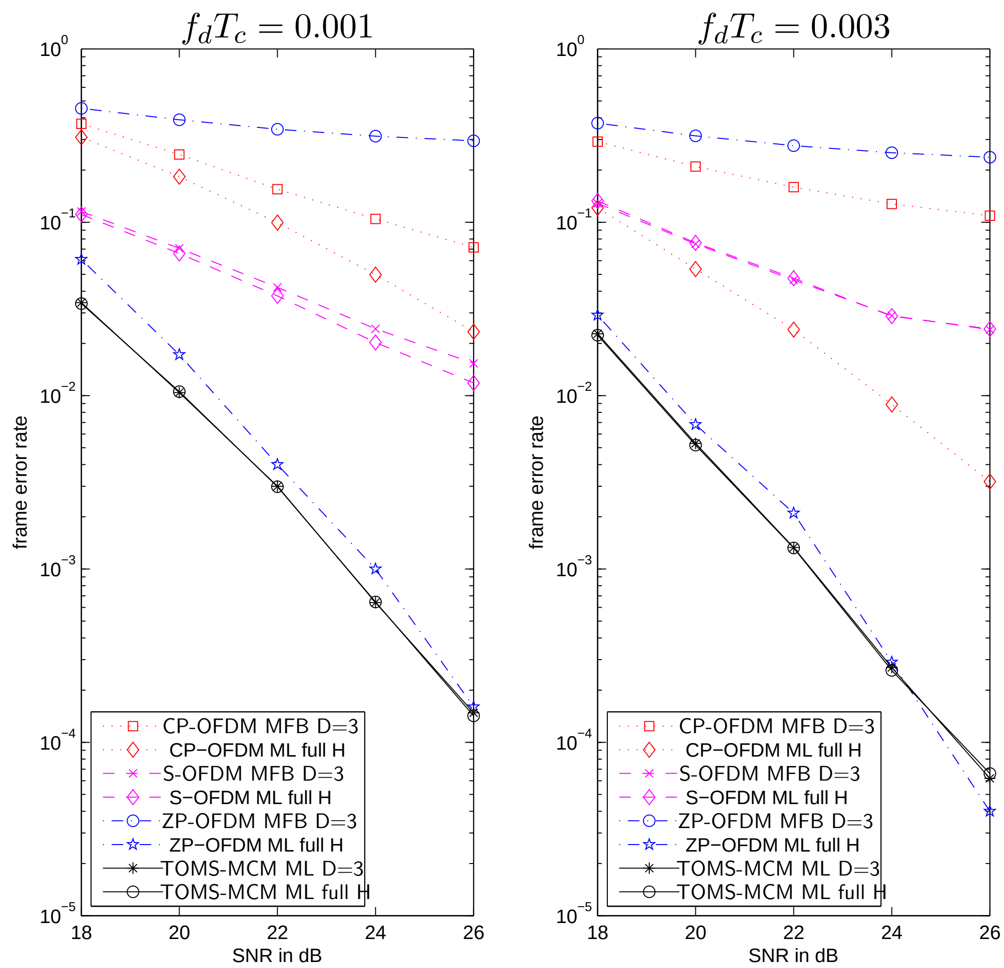
We examine:

- Effect of residual-ICI on the performance of ML estimates.
- Relative performance of various SqDs.
- Relative complexity of various SqDs.
- Effect of imperfect channel knowledge on the performance of various SqDs.

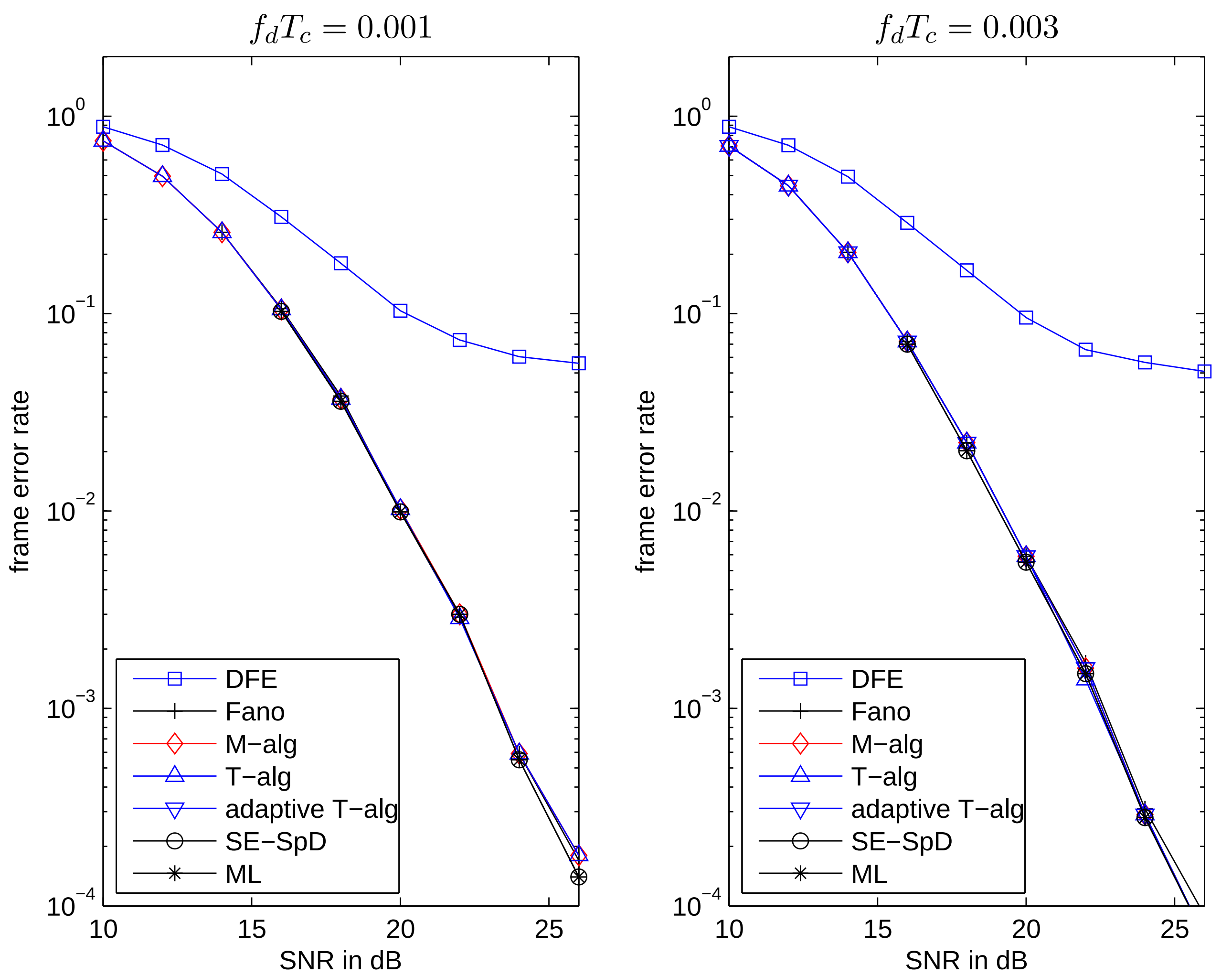
Simulation Setup:

- Uncoded QPSK.
- $N = 64$ subcarriers.
- WSSUS Jakes channel with 16 taps and $f_d T_c \in \{0.001, 0.003\}$, e.g., 10 GHz carrier, 12.5μs delay spread, $\{$138,414$\}$ km/hr.
- CP-, ZP-, and Strohmer-OFDM use $\eta = 0.8$ symbols/sec/Hz, while our TOMS scheme uses $\eta = 1$ symbol/sec/Hz.
- Algorithms employed an ICI radius of $D = 3$.

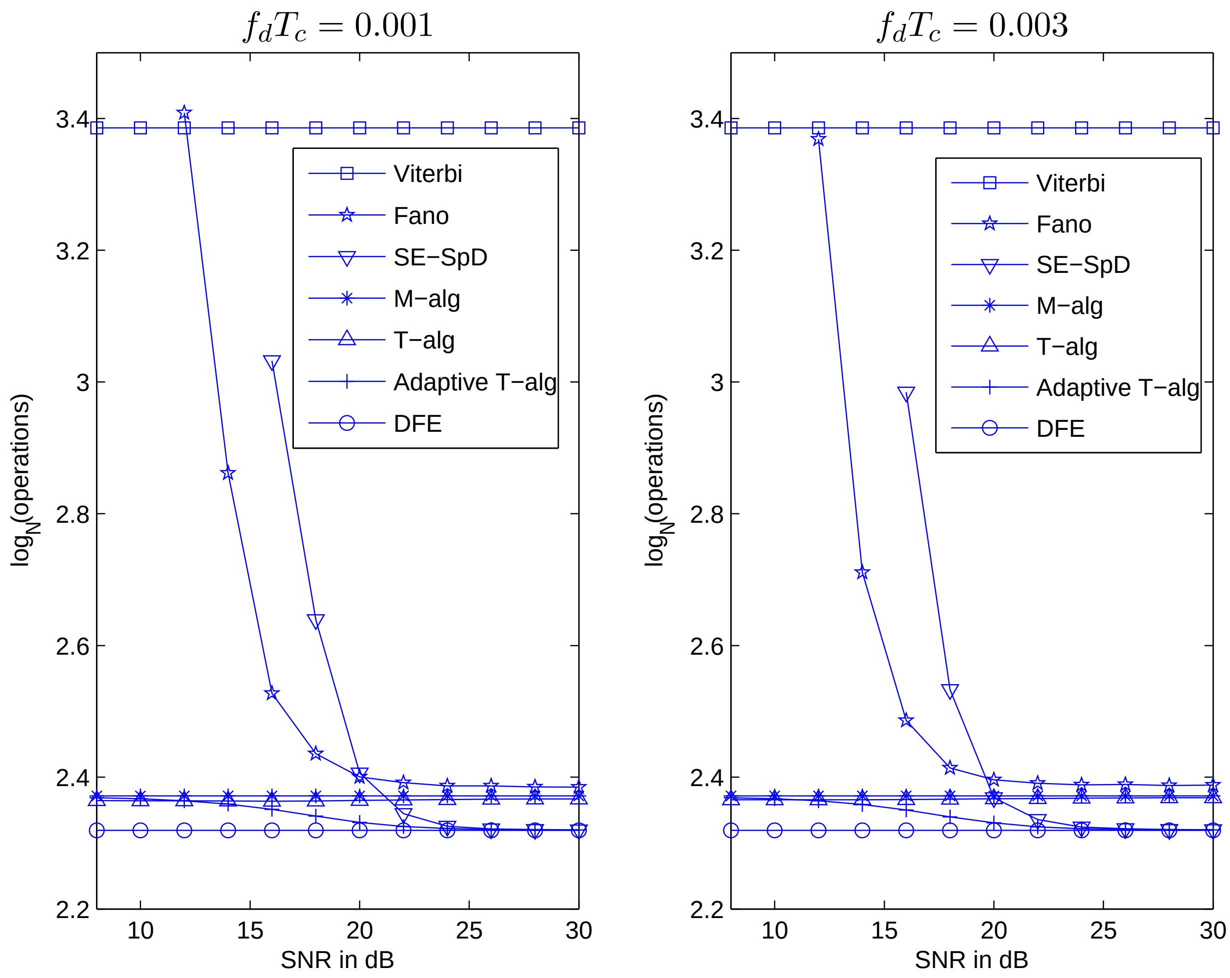
Effect of Residual-ICI/ISI on ML Estimates:



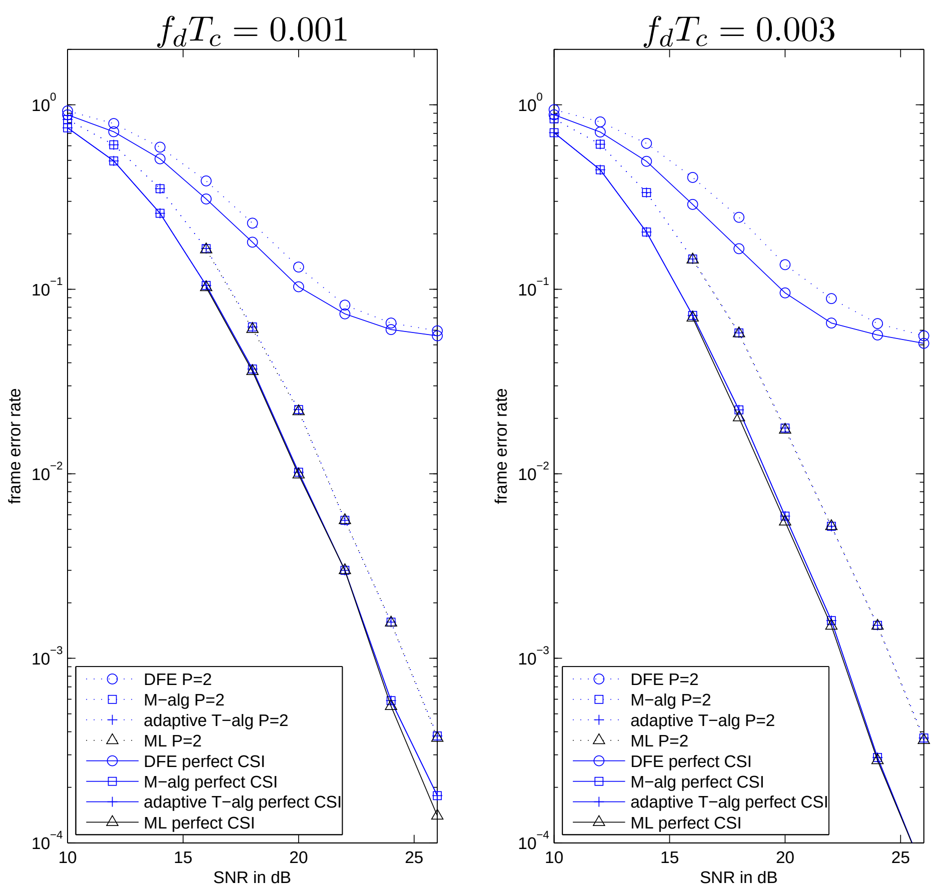
Frame Error Rate:



Average Complexity (MACs/frame):



FER with Imperfect Channel Estimates:



via pilot-aided reduced-rank MMSE estimation of local-ICI coefficients.

Conclusions:

- Pulse shaping can be used to make residual ICI/ISI have a negligible effect on ML performance.
- Pulse shaping can have a degrading effect on MMSE-DFE performance, probably as a result of increasing the sensitivity to error propagation.
- Sequential decoding can yield FERs indistinguishable from that of ML with average complexity on par with MMSE-DFE.
- The banded ICI matrix enables a fast SqD algorithm, but also causes problems for many traditional tree searches (e.g., best-first and depth-first varieties).
- The proposed SqD alg works well with pilot-aided channel estimates.