Non-(Bi)Orthogonal Pulse-Shaped FDM for Doubly-Dispersive Channels

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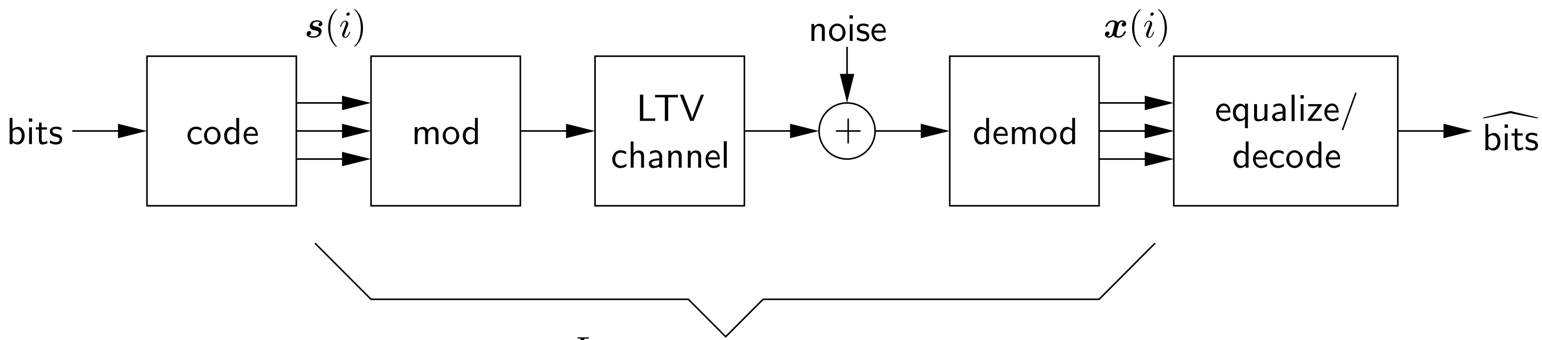
Multicarrier Modulation:



$$\boldsymbol{x}(i) = \sum_{j=-L_{\text{pre}}}^{L_{\text{pst}}} \boldsymbol{\mathcal{H}}(i,j)\boldsymbol{s}(i-j) + \boldsymbol{w}(i)$$

"**LTV MIMO channel**"

- Modulator: multicarrier symbols $\{\boldsymbol{s}(i)\} \to$ waveforms,
- Demodulator: waveforms $\to$ multicarrier observations $\{\boldsymbol{x}(i)\}$.

How should we design modulator/demodulator?

Doubly Dispersive Channel:

- Without dispersion, Nyquist theory specifies a maximum of 1 symbol/sec/Hz for interference-free mod/demod.
- We focus on doubly (i.e., time-frequency) dispersive channels.
- No fixed eigenbasis for these channels, so ISI/ICI is unavoidable in the absence of transmitter channel knowledge.
- Roughly, as symbol/carrier spacings are increased,
  - ISI/ICI decreases (good!), but
  - modulation efficiency decreases (bad!).

$\rightsquigarrow$ *What is the best tradeoff between modulation efficiency and interference suppression?*

(Bi)Orthogonal Signaling

- The traditional solution.
- Main idea:
  - Constrain waveforms for interference-free operation in *non*-dispersive (i.e., trivial) channels.
  - Design waveforms to minimize the ISI/ICI that results from channel dispersion.
- Appeals to the notion of an "approximate eigenbasis" for underspread LTV channels.
- Good interference suppression requires low modulation efficiency (in symbols/sec/Hz).

Non-(Bi)Orthogonal Signaling

The rational:

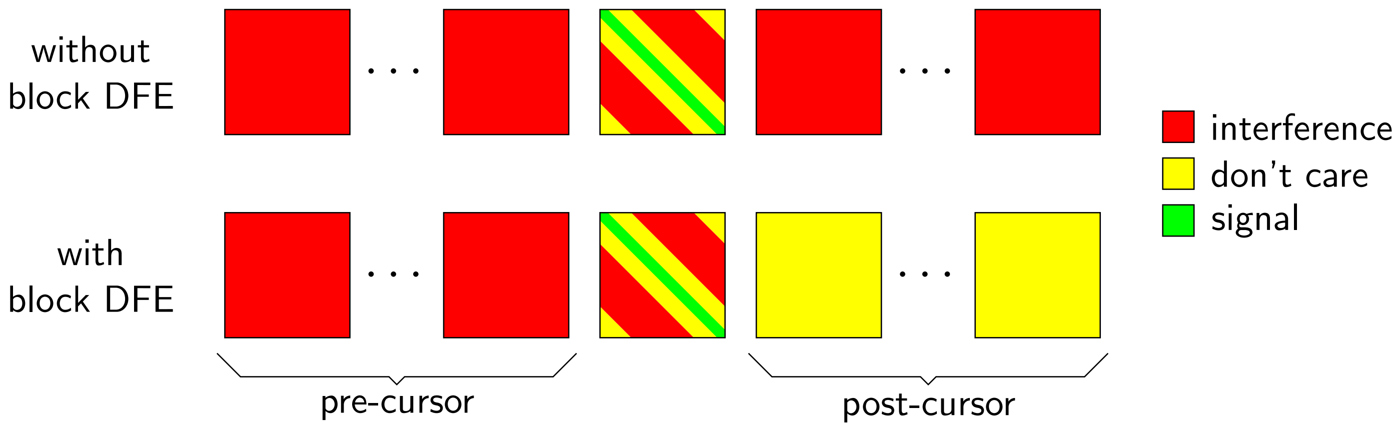
- We don't expect trivial channels, so why design for them?
- We do expect to have an equalizer, so why not leverage it?

Main ideas:

- Shape, rather than suppress, ISI/ICI.
- Design waveforms to yield a target ISI/ICI response that
  - is reachable (i.e., suited to the typical channel),
  - allows low-complexity equalization/decoding.
- An outage capacity analysis suggests that shaping has advantages over suppression. (More later...)

Example: Pulse-Shaped FDM:

- Say we tolerate $\pm D$ subcarriers of neighboring ICI. Target MIMO channel coefs $\{\boldsymbol{\mathcal{H}}(i,-L_{\text{pre}}),\dots,\boldsymbol{\mathcal{H}}(i,L_{\text{pst}})\}$ look like:

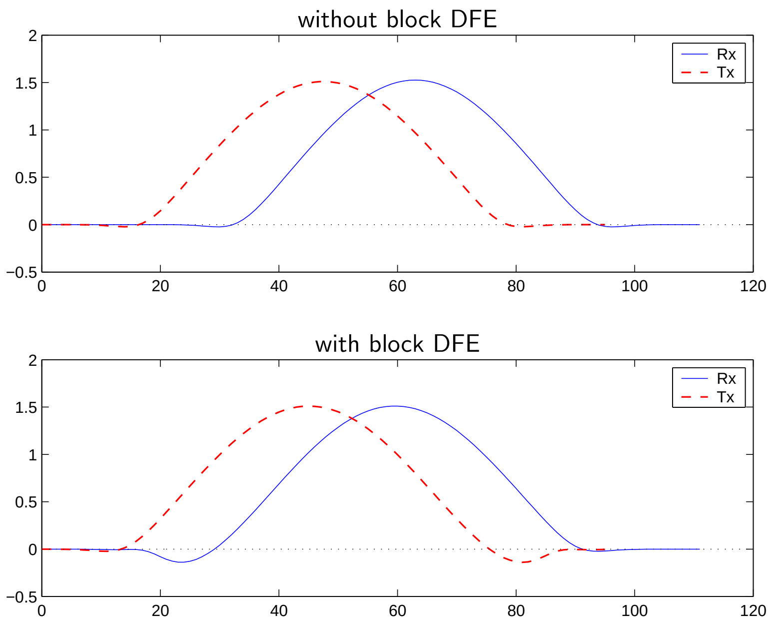


- For transmitter and receiver waveforms that are uniformly modulated versions of pulses $a(t)$ and $b(t)$, respectively, can obtain SINR-maximizing pulses by alternating between two generalized eigenvalue problems. (Requires knowledge of Doppler spectrum, power-delay profile, and SNR.) Allows efficient FFT-based modulation and demodulation, i.e., OFDM complexity.

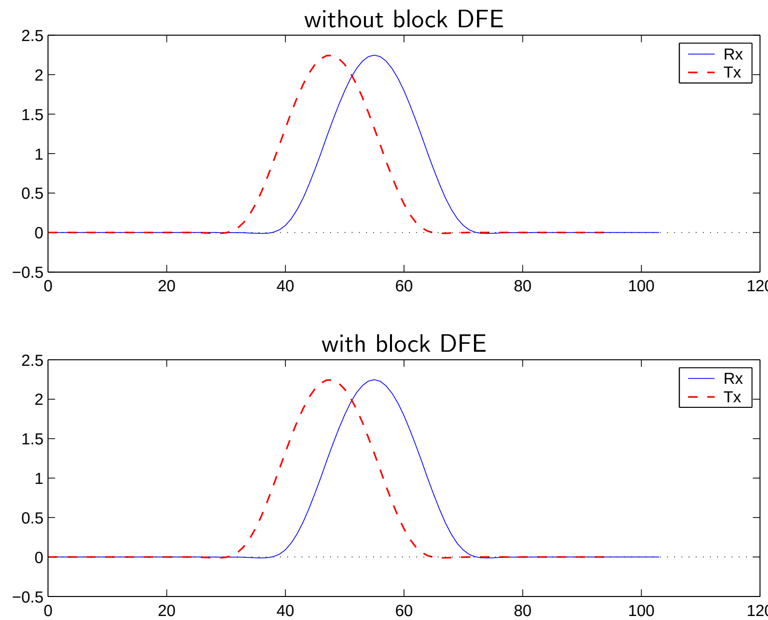
Typical Max-SINR Pulse Shapes:

$$\mathsf{BW}_{\mathsf{total}} = B \text{ Hz}, \quad N = 64 \text{ carriers}, \quad T_s = \frac{N}{B} \quad \longrightarrow \quad 1 \text{ sym/sec/Hz}.$$

$$T_{\mathsf{ISI}} = \frac{T_s}{2}, \ \mathrm{SNR} = 20\mathrm{dB}, \ \frac{f_{\mathsf{d}}}{B} = 0.03:$$



$$T_{\mathsf{ISI}} = \frac{T_s}{4}, \ \mathrm{SNR} = 5\mathrm{dB}, \ \frac{f_{\mathsf{d}}}{B} = 0.1:$$



System Design:

- Traditionally, symbol interval T_s and carrier spacing B/N chosen to minimize ISI/ICI (at the cost of modulation efficiency).
- Now we *tolerate* ISI/ICI. So how do we choose the following?
  - D: target ICI radius.
  - N: number of subcarriers.
  - $\frac{N}{BT_s}$: modulation efficiency (symbols/sec/Hz).
- Assuming the use of powerful coding, with delay constraints at the decoder, outage capacity is an appropriate performance measure.

Outage Capacity:

- Definition of outage capacity C_o via probability P_o:

$$P_o \;:=\; \Pr\big\{\mathcal{I}^{(j)} < C_o\big\}$$

- Example setup with $M = 2, L_{\text{pre}} = 1, L_{\text{pst}} = 1$:

$$\begin{bmatrix} \boldsymbol{x}^{(1)} \\ \boldsymbol{x}^{(0)} \end{bmatrix} = \left[\begin{array}{c|cc|c} \boldsymbol{\mathcal{H}}(1,-1) & \boldsymbol{\mathcal{H}}(1,0) & \boldsymbol{\mathcal{H}}(1,1) & \\ & \boldsymbol{\mathcal{H}}(0,-1) & \boldsymbol{\mathcal{H}}(0,0) & \boldsymbol{\mathcal{H}}(0,1) \end{array}\right] \left[\begin{array}{c} \boldsymbol{s}(2) \\ \hline \boldsymbol{s}(1) \\ \boldsymbol{s}(0) \\ \hline \boldsymbol{s}(-1) \end{array}\right] + \begin{bmatrix} \boldsymbol{w}^{(1)} \\ \boldsymbol{w}^{(0)} \end{bmatrix}$$

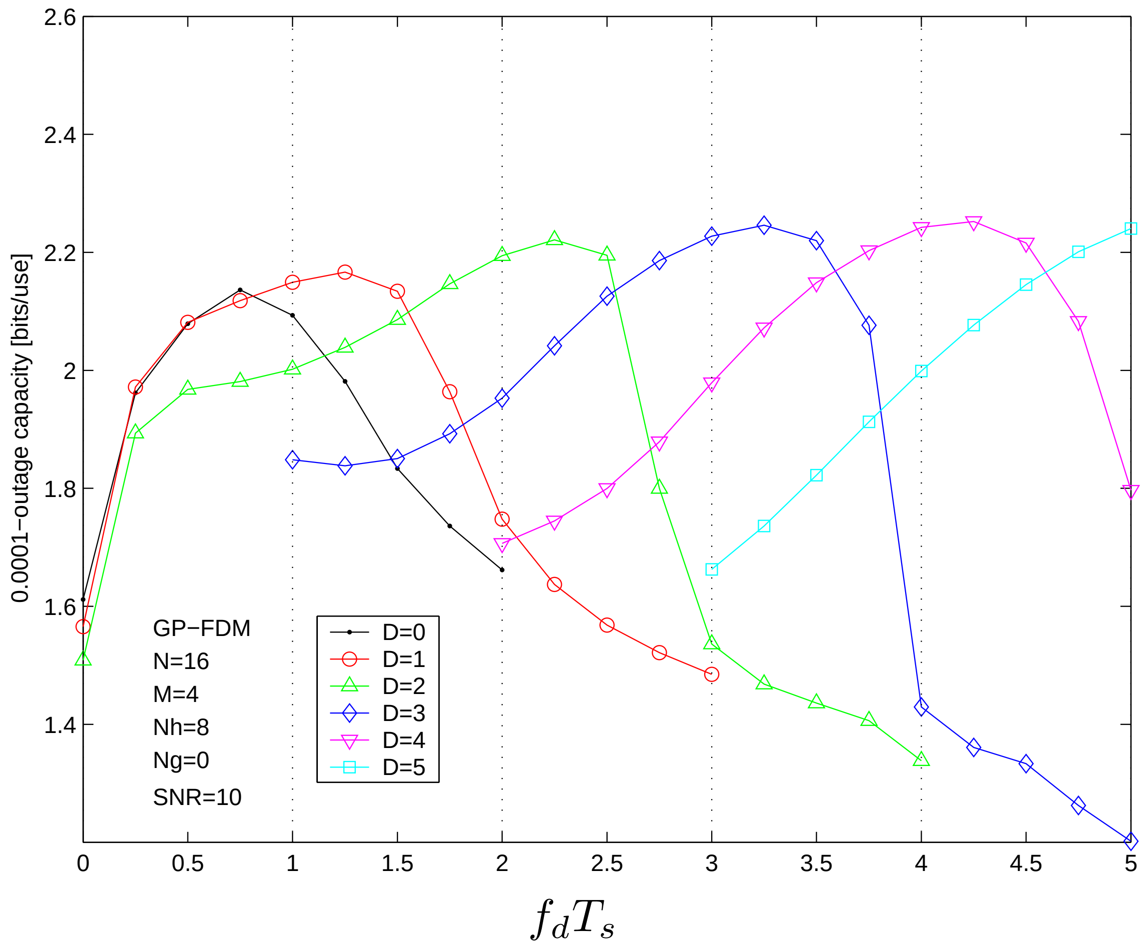
$$\underbrace{\begin{bmatrix} \boldsymbol{x}^{(1)} \\ \boldsymbol{x}^{(0)} \end{bmatrix}}_{\boldsymbol{x}^{(0)}} = \underbrace{\begin{bmatrix} \boldsymbol{\mathcal{H}}(1,0) & \boldsymbol{\mathcal{H}}(1,1) \\ \boldsymbol{\mathcal{H}}(0,-1) & \boldsymbol{\mathcal{H}}(0,0) \end{bmatrix}}_{\boldsymbol{\mathcal{H}}^{(0)}} \underbrace{\begin{bmatrix} \boldsymbol{s}(1) \\ \boldsymbol{s}(0) \end{bmatrix}}_{\boldsymbol{s}^{(0)}} + \underbrace{\begin{bmatrix} \boldsymbol{\mathcal{H}}(1,-1) & \\ & \boldsymbol{\mathcal{H}}(0,1) \end{bmatrix} \begin{bmatrix} \boldsymbol{s}(2) \\ \boldsymbol{s}(-1) \end{bmatrix} + \begin{bmatrix} \boldsymbol{w}^{(1)} \\ \boldsymbol{w}^{(0)} \end{bmatrix}}_{\boldsymbol{v}^{(0)}}$$

- Mutual info (bits/sec/Hz) between Gaussian $\boldsymbol{s}^{(j)}$ and $\boldsymbol{x}^{(j)}$

$$\mathcal{I}^{(j)} \;=\; \frac{1}{MN_s} \log_2 \det\big(\boldsymbol{I}_{MN} + \boldsymbol{\mathcal{H}}^{(j)H} \boldsymbol{R}_v^{-1} \boldsymbol{\mathcal{H}}^{(j)}\big)$$

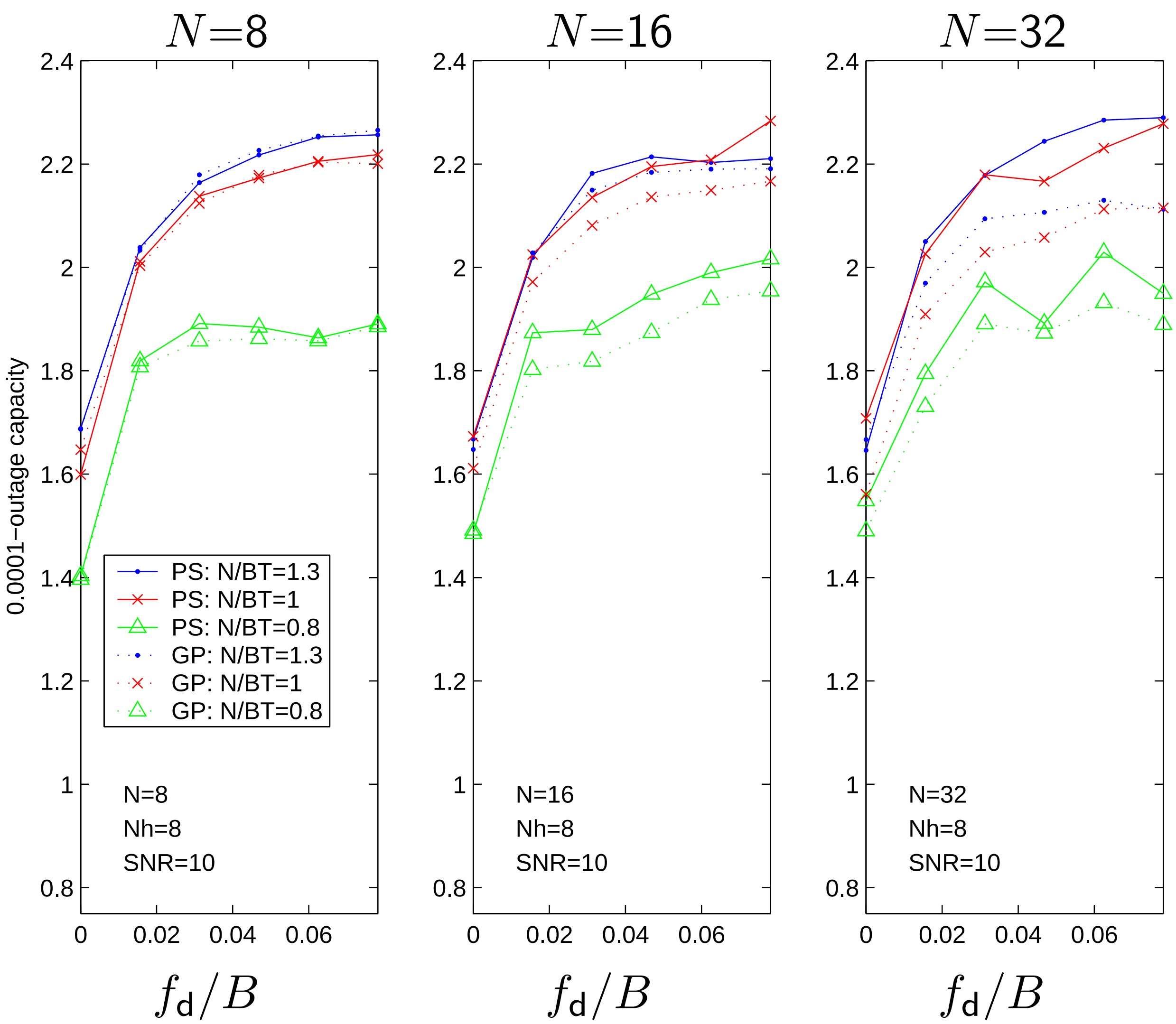
where $N_s = BT_s$ and M is # of m.c. symbols in a code block.

Outage Capacity vs $f_{\mathrm{d}}T_s$ for various D:



$\Rightarrow$ Max-SINR pulse designs based on an ICI radius of $\approx f_{\mathrm{d}}T_s$ have a capacity advantage at higher Dopplers!

Capacity vs f_d/B for various $\{N, \frac{N}{BT_s}\}$:

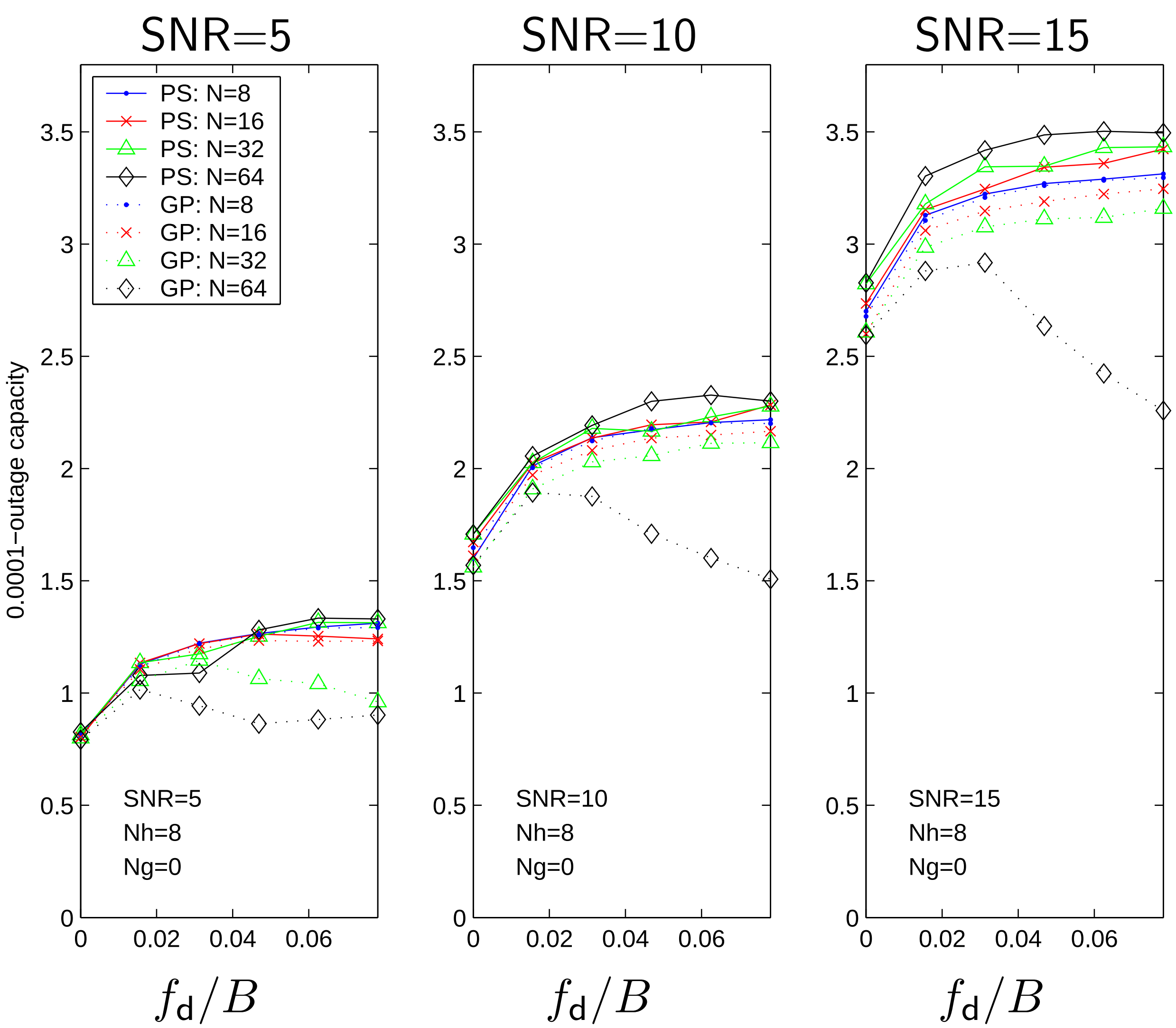


Relative to $\eta_\mathsf{mod} = 1\ldots$

- small gain from overloading $(N/BT_s = 1.3)$,
- larger penalty from redundancy $(N/BT_s = 0.8)$.

for (almost) all $\{N, f_\mathsf{d}/B, \mathsf{SNR}\}$.

Capacity vs f_{d}/B for various $\{\textbf{SNR}, N\}$:



Suggests. . .

- PS-FDM and GP-FDM similar for $N = 8$,
- capacity slightly increases with N for PS-FDM,
- capacity decreases with N for GP-FDM.

Conclusions:

- Considered interference shaping, rather than interference suppression, to design multicarrier signaling waveforms for doubly dispersive channels.
- Neighboring-ICI can be mitigated using low-complexity iterative equalization/decoding (described elsewhere).
- Postcursor-ISI mitigated using block decision feedback.
- Used to design max-SINR pulse shapes for FDM system, allowing FFT-based transmitter/receiver.
- Outage-capacity analysis suggests performance advantages over interference-suppressing designs in coded systems.