

# Rate Adaptation via Link-Layer Feedback for Goodput Maximization over a Time-Varying Channel

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**Abstract**—The variable nature of the wireless channel may cause the quality of service to be intolerable for certain applications. To combat channel variability, we consider rate adaptation at the physical layer. We build an adaptive communication system based on uncoded QAM in which the available information on the channel state is obtained using the mere packet-level ACK/NACK sequence. Our system chooses the constellation size that maximizes the expected packet level goodput for every single packet. Our simulations show that our system achieves a goodput, reasonably close to the highest possible goodput achievable with full-feedback on Rayleigh-fading Markov channels.<sup>1</sup>

## I. INTRODUCTION

Channel variability is common to all wireless communications due to factors such as fading, mobility and multiuser interference. One way to combat the detrimental effects of variability is using rate adaptation. The idea is that, based on the measured channel state, the transmitter alters the size of the signal constellation to try to maximize the amount of data transferred without an error. Consequently the channel variability will be experienced at the higher layers in the form of *variable data rate*, rather than *variable error rate*.

Rate adaptation has been addressed mainly in two different perspectives. The first group (e.g., [1], [2] and [3]) assumes specific channel models and focuses on the design of various adaptive codes based on these models. Channel state information (CSI) is obtained using physical layer symbols called pilots. Second group (e.g., [4], [5], [6] and [7]) proposes rate adaptation algorithms developed based on the some practical experience gained in a certain environment and a specific application. The main focus of these papers is to propose modifications to the existing 802.11 MAC layer and present some experimentally demonstrated improvements relative to the predecessors. CSI is acquired based on the packet-level ACK/NACK sequence, available at the link-layer.

Our system, described in Section II, combines the ideas of the two groups: Like the latter group, we assume only link-level CSI in the form of packet ACK/NACKs. On the other hand, like to first group, we assume a certain channel model (Rayleigh fading, first order Markov) and our system finds the *optimal constellation size*, which maximizes the expected goodput for each packet. For that purpose, our system uses

an algorithm that updates the constellation size iteratively, using the most recent ACK packet. Each iteration involves the numerical evaluation of an integral. To reduce the complexity, we propose a modified algorithm which processes the ACK sequence in batches and hence the integration is needed for groups of ACK packets, instead of every single one. The details of the algorithm can be found in Section III.

We evaluate the performance of our system in Section IV. We show that, the goodput achieved by our system is within 8% of the highest goodput achievable with full CSI. Moreover, the improvement over the best non-adaptive algorithm can be as high as 30%. This is somewhat surprising due to the fact that the CSI contained in a single acknowledgement, no more than 1 bit per packet, is extremely coarse. We also analyze the goodput-complexity trade-off as we evaluate the performance of the modified algorithm. Finally, we wrap up with some conclusions and future extensions in Section V.

## II. PROBLEM SETUP

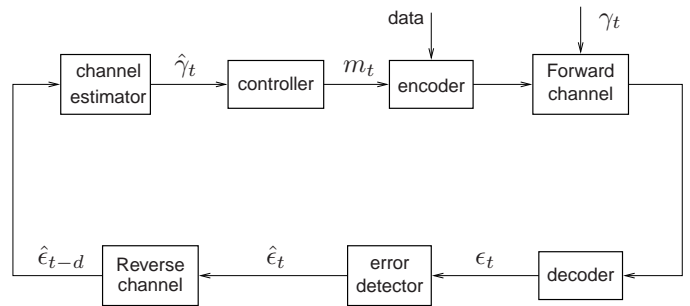


Fig. 1. System Model.

Figure 1 shows the system model. The model assumes that the transmitter can obtain delayed feedback on the reliability of previously transmitted packets and that this feedback can be used to optimize future transmissions. In particular, the transmitter uses the feedback information to predict the future SNR characteristics, from which it adapts the future transmission parameters. In this paper, we focus on the case where feedback comes in the form of binary packet acknowledgments (i.e., ACK/NACKs) and the transmission parameter is (uncoded) constellation size, though our approach could be

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straightforwardly extended to the adaptation of transmit power and/or code rate.

The time-varying wireless channel is modeled by an SNR process  $\{\gamma_t\}$ , where  $t$  denotes the packet index and  $\gamma_t \geq 0$ . Since the transmission power is fixed,  $\gamma_t$  is an exogenous process that does not depend on the transmission parameters. For QAM constellation size  $m$  and SNR  $\gamma$ , the symbol error rate is known to be [8, p. 280]

$$\epsilon = 1 - \left(1 - 2 \left(1 - \frac{1}{\sqrt{m}}\right) Q \left(\sqrt{\frac{3\gamma}{m-1}}\right)\right)^2. \quad (1)$$

Assuming that the SNR and constellation size are fixed over a packet of  $r$  symbols, the packet error rate (for the  $t$ -th packet) then equals

$$\epsilon_t = 1 - \left(1 - 2 \left(1 - \frac{1}{\sqrt{m_t}}\right) Q \left(\sqrt{\frac{3\gamma_t}{m_t-1}}\right)\right)^{2r}. \quad (2)$$

Defining the instantaneous goodput  $G(m_t, \gamma_t)$  as the expected number of bits-per-symbol communicated without error in a packet with constellation size  $m_t$  and SNR  $\gamma_t$ , we have

$$\begin{aligned} G(m_t, \gamma_t) &= (1 - \epsilon_t) \log m_t & (3) \\ &= \left(1 - 2 \left(1 - \frac{1}{\sqrt{m_t}}\right) Q \left(\sqrt{\frac{3\gamma_t}{m_t-1}}\right)\right)^{2r} \\ &\quad \times \log m_t. & (4) \end{aligned}$$

From (4), it can be seen that there is a one-to-one correspondence between constellation size  $m_t$  and goodput for a fixed SNR  $\gamma_t$ . Thus, if the SNR was known perfectly, then goodput can be maximized by appropriate choice of constellation size. Figure 2 plots instantaneous goodput versus SNR and constellation size for the case of  $r = 100$  symbols per packet, highlighting the goodput-maximizing constellation size as a function of SNR. From the figure, it can be seen that (for this  $r$ ) the goodput maximizing value of  $\log m_t$  increases approximately linearly with  $\log \gamma_t$  when  $\gamma_t > 15$  dB, as does the maximum goodput.

Suppose now that we are interested in choosing the rate sequence  $\{m_t\}_{t=1}^l$  that maximizes the finite horizon goodput  $\sum_{t=1}^l G(m_t, \gamma_t)$ . Though in practice we do not expect to know the SNR realization  $\{\gamma_t\}_{t=1}^l$  (especially at the time of choosing  $\{m_t\}_{t=1}^l$ ), we might know its statistics. In this case we consider the problem of maximizing the expected goodput  $E\{\sum_{t=1}^l G(m_t, \gamma_t)\} = \sum_{t=1}^l E\{G(m_t, \gamma_t)\}$ , for which it can be seen that the marginal SNR statistics suffice. Under mild assumptions,  $E\{G(m_t, \gamma_t)\}$  is maximized by a unique  $m_t$ , making the solution straightforward. But it is important to realize that the expected goodput is reduced by uncertainty in the SNR, since

$$\max_{m_t} E\{G(m_t, \gamma_t)\} \leq \max_{m_t} G(m_t, E\{\gamma_t\}). \quad (5)$$

Thus we are encouraged in reducing uncertainty in  $\gamma_t$  by any means possible.

In this paper, we consider the problem of choosing  $\{m_t\}_{t=1}^l$  to maximize  $\sum_{t=1}^l E\{G(m_t, \gamma_t)\}$ , where  $\{\gamma_t\}_{t=1}^l$  is random

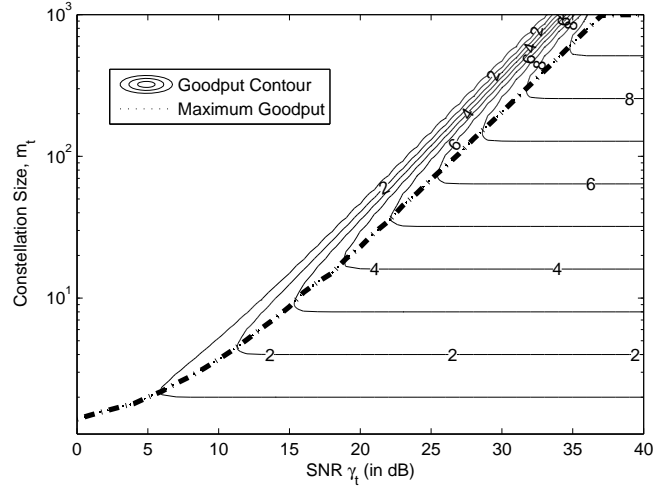


Fig. 2. Goodput contours versus SNR and constellation size for packet size  $r = 100$ . Also shown is the goodput maximizing constellation size as a function of SNR.

with known statistics, and where strictly causal estimates of the packet error rate  $\epsilon_t$  are available. These error rate estimates offer the potential to reduce the uncertainty of  $\gamma_t$  and thus to increase goodput. With causal feedback, however, the optimization of  $\{m_t\}$  becomes more complicated because  $m_t$  affects not only the current expected goodput  $E\{G(m_t, \gamma_t)\}$  but also the statistics of future errors, and thus the future expected goodput.

### III. A GREEDY RATE ADAPTATION ALGORITHM

Notice that, for the problem defined in the previous paragraph, the search for the optimal  $\{m_t\}_{t=1}^l$  appears to be NP-hard. Thus, for practical horizons  $l$ , optimal rate selection appears infeasible and suboptimal approaches are of interest. Towards this aim, we propose a greedy scheme which considers the effect of constellation size on current, but not future, expected goodput. One might also pursue a scheme which considers a few, but not all, future goodputs when choosing the current constellation size. However, numerical experiments in Section IV suggest that our simple greedy scheme extracts a large fraction of the goodput gain that can be achieved under the causal feedback constraint.

#### A. Packet-Rate Algorithm

For simplicity, we first describe our scheme assuming that the transmitter can adapt the constellation size on a per-packet basis. Later we address the more practical scenario where the transmitter can only update the constellation size once per block of  $n$  packets. Assuming a feedback delay of  $d \geq 1$  packets, our greedy rate-selection scheme is described by (6)-(7),

$$\hat{m}_{t|t-d} = \operatorname{argmax}_{m_t \in M} \bar{G}_{t|t-d}(m_t) \quad \text{for } t = 1, \dots, l. \quad (6)$$

$$\bar{G}_{t|t-d}(m_t) = \int G(m_t, \gamma_t) p(\gamma_t | \hat{\epsilon}_{t-d}, \mathbf{m}_{t-d}) d\gamma_t, \quad (7)$$

where  $\hat{\epsilon}_{t-d} = [\hat{\epsilon}_1, \dots, \hat{\epsilon}_{t-d}]$ ,  $\mathbf{m}_{t-d} = [m_1, \dots, m_{t-d}]$ , and  $M$  denotes the set of allowed constellation sizes. Under the assumption of Markov SNR variation and conditionally independent error-rate estimates, we derive a recursive algorithm for the greedy rate assignment (6).

Expanding (7) via Bayes theorem, we get

$$\begin{aligned} & p(\gamma_t | \hat{\epsilon}_{t-d}, \mathbf{m}_{t-d}) \\ &= \int p(\gamma_t | \gamma_{t-d}, \hat{\epsilon}_{t-d}, \mathbf{m}_{t-d}) p(\gamma_{t-d} | \hat{\epsilon}_{t-d}, \mathbf{m}_{t-d}) d\gamma_{t-d} \\ &= \int p(\gamma_t | \gamma_{t-d}) p(\gamma_{t-d} | \hat{\epsilon}_{t-d}, \mathbf{m}_{t-d}) d\gamma_{t-d}, \end{aligned} \quad (8)$$

where, with conditionally independent error estimates (i.e.,  $p(\hat{\epsilon}_t | \epsilon_t, \hat{\epsilon}_{t-1}) = p(\hat{\epsilon}_t | \epsilon_t)$ ), we get

$$\begin{aligned} & p(\gamma_{t-d} | \hat{\epsilon}_{t-d}, \mathbf{m}_{t-d}) \\ &= p(\gamma_{t-d} | \hat{\epsilon}_{t-d}, \hat{\epsilon}_{t-d-1}, \mathbf{m}_{t-d}) \\ &= \frac{p(\hat{\epsilon}_{t-d} | \gamma_{t-d}, \hat{\epsilon}_{t-d-1}, \mathbf{m}_{t-d}) p(\gamma_{t-d} | \hat{\epsilon}_{t-d-1}, \mathbf{m}_{t-d})}{\int p(\hat{\epsilon}_{t-d} | \gamma'_{t-d}, \hat{\epsilon}_{t-d-1}, \mathbf{m}_{t-d}) p(\gamma'_{t-d} | \hat{\epsilon}_{t-d-1}, \mathbf{m}_{t-d}) d\gamma'_{t-d}} \\ &= \frac{p(\hat{\epsilon}_{t-d} | \gamma_{t-d}, \mathbf{m}_{t-d}) p(\gamma_{t-d} | \hat{\epsilon}_{t-d-1}, \mathbf{m}_{t-d-1})}{\int p(\hat{\epsilon}_{t-d} | \gamma'_{t-d}, \mathbf{m}_{t-d}) p(\gamma'_{t-d} | \hat{\epsilon}_{t-d-1}, \mathbf{m}_{t-d-1}) d\gamma'_{t-d}}. \end{aligned} \quad (9)$$

Similar to (8), we can also write

$$\begin{aligned} & p(\gamma_{t-d+1} | \hat{\epsilon}_{t-d}, \mathbf{m}_{t-d}) \\ &= \int p(\gamma_{t-d+1} | \gamma_{t-d}, \hat{\epsilon}_{t-d}, \mathbf{m}_{t-d}) \\ &\quad \times p(\gamma_{t-d} | \hat{\epsilon}_{t-d}, \mathbf{m}_{t-d}) d\gamma_{t-d} \\ &= \int p(\gamma_{t-d+1} | \gamma_{t-d}) p(\gamma_{t-d} | \hat{\epsilon}_{t-d}, \mathbf{m}_{t-d}) d\gamma_{t-d}. \end{aligned} \quad (10)$$

Equations (7)–(10) lead to the following recursive solution of the rate assignment problem (6). Assuming the availability of  $p(\gamma_{t-d} | \hat{\epsilon}_{t-d-1}, \mathbf{m}_{t-d-1})$  at the start of iteration  $t$ , we propose the following procedure for  $t = d+1, \dots, l$  (i.e., for the packet indices where the feedback  $\hat{\epsilon}_{t-d}$  is available):

- 1) Measure  $\hat{\epsilon}_{t-d}$  and compute<sup>2</sup> the distribution  $p(\gamma_{t-d} | \hat{\epsilon}_{t-d}, \mathbf{m}_{t-d})$  using (9),
- 2) Calculate  $p(\gamma_t | \hat{\epsilon}_{t-d}, \mathbf{m}_{t-d})$  using the Markov prediction step (8),
- 3) Choose  $m_t$  to maximize  $\bar{G}_{t|t-d}(m_t)$  via (7),
- 4) If<sup>3</sup>  $d > 1$ , then compute  $p(\gamma_{t-d+1} | \hat{\epsilon}_{t-d}, \mathbf{m}_{t-d})$  via (10) for use in the next iteration.

For the indices  $t = 1, \dots, d$  (i.e., before feedback is available), we set  $p(\gamma_t | \hat{\epsilon}_{t-d}, \mathbf{m}_{t-d}) = p(\gamma_t)$ , i.e., the prior distribution on SNR.

### B. Block-Rate Algorithm

Since it may be impractical for the transmitter to adapt the constellation size on a per-packet basis, we now extend our greedy scheme to one where the constellation size is adapted on a per-block basis, using  $n$  to denote the number of packets per block. To do this, we treat the SNR, constellation size, and

<sup>2</sup>Notice that, with known  $p(\hat{\epsilon}_t | \epsilon_t)$ , the quantity  $p(\hat{\epsilon}_t | \gamma_t, m_t)$  can be calculated using (2).

<sup>3</sup>Notice that, if  $d = 1$ , then  $p(\gamma_{t-d+1} | \hat{\epsilon}_{t-d}, \mathbf{m}_{t-d})$  was already computed in step 2).

error estimates as if they were *constant* over the block, yielding a greedy algorithm that would operate as previously described, but at the block level (i.e., where the subscript would refer to the block index). To avoid confusion, however, we continue to interpret all subscripts as packet indices in the sequel. Thus, in the block algorithm,  $\hat{\epsilon}_t$  refers to an estimate of the *average* packet error rate over the  $n$  packets in the  $\lfloor \frac{t}{n} \rfloor$ -th block, while  $\gamma_t$  refers to the SNR for the packet in the *middle* of the  $\lfloor \frac{t}{n} \rfloor$ -th block.

As the block size  $n$  increases, we expect the packet error rate estimate  $\hat{\epsilon}_t$  to get more accurate (since it is estimated from, e.g.,  $n$  ACK/NACKs), the SNR model to get less accurate (since a block-fading approximation is being applied to a process that is continuously fading), and the per-packet computational complexity of the algorithm to decrease.

It is important to note that the block-rate modification we propose here is suboptimal in the sense that we could have chosen to predict the SNR for each individual packet in a given block, rather than predicting only the SNR of the packet in the middle of the block. Likewise, we could have chosen individual constellation sizes for each packet in a given block rather than a single constellation size for all packets in the block. Clearly, if the SNR predictions varied across the block, then the optimal constellation sizes could vary as well. However, the joint optimization of intra-block constellation sizes block appears prohibitively complex, and thus goes against our primary motivation for a block-level algorithm, i.e., simplicity.

## IV. NUMERICAL RESULTS

We now describe the results of numerical experiments for which we assumed a particular error estimation scheme and particular SNR process, both described below.

### A. Setup

We assumed an ARQ feedback network wherein the transmitter obtains an ACK/NACK for each of the packets it transmits. Thus, in a block of  $n$  packets, there are  $n$  ACK/NACKs. Under these assumptions, the minimum-variance unbiased estimate  $\hat{\epsilon}_t$  of the (average) packet error rate over the  $\lfloor \frac{t}{n} \rfloor$ -th block can be computed by a simple arithmetic average of the  $n$  ACK/NACKs over the block, after assigning 0 to an ACK and 1 to a NACK. Since, for true  $\epsilon_t$ , the number of NACKs in the block is Binomial( $n, \epsilon_t$ ), it follows that error estimate  $\hat{\epsilon}_t$  obeys

$$p(\hat{\epsilon}_t = \frac{k}{n} | \epsilon_t) = \begin{cases} \binom{n}{k} \epsilon_t^k (1 - \epsilon_t)^{n-k} & \text{for } k = 0, \dots, n \\ 0 & \text{else.} \end{cases} \quad (11)$$

Recall that we can substitute (2) into  $p(\hat{\epsilon}_t | \epsilon_t)$  to obtain  $p(\hat{\epsilon}_t | \gamma_t, m_t)$ .

The following Markov model for packet-rate SNR evolution was assumed. While a Gauss-Markov random-walk model for  $\gamma_t$  (i.e.,  $\gamma_t = (1 - \alpha)\gamma_{t-1} + \alpha w_t$  for  $\alpha \in [0, 1)$  and i.i.d.  $w_t \sim \mathcal{N}(0, \sigma^2)$ ) would have been very convenient, it gives no guarantee that  $\gamma_t \geq 0$ , as required for a meaningful SNR. Thus, we found it more appropriate to employ a Gauss-Markov

model for a “channel gain”  $g_t$ , from which the SNR  $\gamma_t$  follows as a (scaled) squared magnitude:

$$g_t = (1 - \alpha)g_{t-1} + \alpha w_t \quad (12)$$

$$\gamma_t = K|g_t|^2, \quad (13)$$

In (12), we assume circular Gaussian unit-variance white noise  $\{w_t\}$ , so that  $g_t \in \mathbb{C}$ . It can be shown that, in the steady state,  $\gamma_t$  follows an exponential distribution with mean value  $\frac{2K\alpha}{2-\alpha}$ . Note that the parameters  $\alpha$  and  $K$  control the rate of channel variation as well as the average SNR. To evaluate  $p(\gamma_t|\gamma_{t-d})$ , we first notice from (12) that

$$g_t = (1 - \alpha)^d g_{t-d} + \alpha \sum_{j=0}^{d-1} (1 - \alpha)^j w_{t-j}, \quad (14)$$

where  $\sum_{j=0}^{d-1} (1 - \alpha)^j w_{t-j} \sim \mathcal{CN}(0, \frac{1}{1 - (1 - \alpha)^2} (1 - (1 - \alpha)^{2d}))$ . It is then shown in the Appendix that

$$p(\gamma_t|\gamma_{t-d}) = \frac{2 - \alpha}{2K\alpha(1 - (1 - \alpha)^{2d})} \times \exp\left(\frac{-(\gamma_t + (1 - \alpha)^{2d}\gamma_{t-d})(2 - \alpha)}{2K\alpha(1 - (1 - \alpha)^{2d})}\right) \times I_0\left(\frac{(1 - \alpha)^d \sqrt{\gamma_t \gamma_{t-d}}(2 - \alpha)}{K\alpha(1 - (1 - \alpha)^{2d})}\right). \quad (15)$$

## B. Results

Numerical experiments were conducted to investigate the steady state performance of the greedy algorithm from Section III relative to two reference schemes. The so-called *fixed-rate* scheme chooses the constellation size that maximizes expected goodput according to the prior SNR distribution. Since the prior doesn’t change, the constellation size remains fixed. Given our system model, this fixed-rate scheme is the optimal scheme in the absence of feedback. The so-called *causal genie* scheme adapts the constellation size to maximize expected goodput under perfect (though strictly causal) SNR feedback. Note that the causal genie still succumbs to SNR prediction error, especially as the feedback delay or block size increases. Given our system model, the causal genie upper bounds the performance of *any* scheme that maximizes expected goodput under strictly causal feedback.

In our experiments, the channel parameters were chosen so that  $\alpha = 0.01$  and  $E\{\gamma_t\} = 10$ , and the steady-state goodput was calculated by averaging the instantaneous goodputs achieved by the various algorithms over 200 packets, each consisting of 100 symbols, for 500 channel realizations. To suppress the initial transient, we initialized the greedy algorithm at the goodput-maximizing constellation size. For the greedy and causal genie algorithms, we allowed constellation sizes between 1.1 and 12 in steps of 0.1.

Figure 3 plots steady-state goodput versus feedback delay  $d$  for block size  $n = 1$ . There it can be seen that, at low delay (i.e.,  $d = 1$ ), the causal genie yields a 40% increase over the steady-state goodput attained by the fixed-rate algorithm. As the delay increases, however, the predicted SNR distribution converges to the prior, so that after 100 packets the causal

genie performs essentially the same as the fixed-rate algorithm. The goodput gain attained by greedy algorithm can be seen to be closer to that of the genie than to that of the fixed-rate algorithm. Thus, we conclude that the simple greedy scheme captures a dominant fraction (i.e.,  $\approx 73\%$ ) of the goodput gain achievable under causal feedback.

Figure 4 plots steady-state goodput versus block size  $n$  for delay  $d = 1$ . For most block sizes, the greedy algorithm again performs closer to the causal genie than to the fixed-rate algorithm, implying that the greedy algorithm recovers a dominant portion of the goodput gain achievable under the causal feedback constraint. As the block size approaches 100, though, the performances of the adaptive schemes converge to that of the fixed-rate scheme, again due to the fundamental impossibility of predicting SNR far in the future.

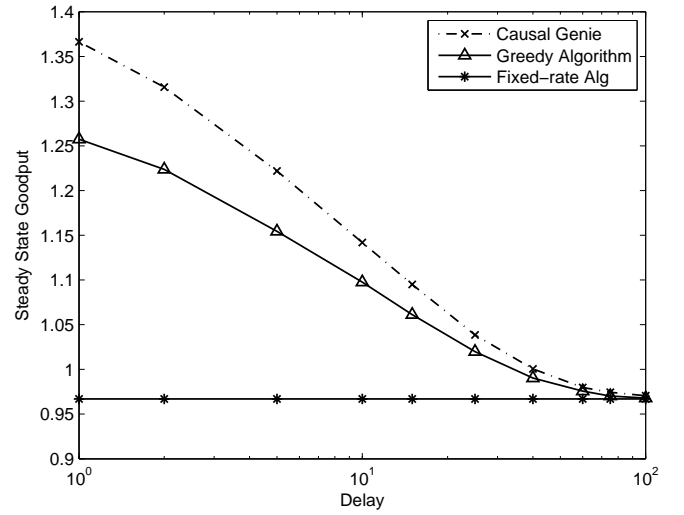


Fig. 3. Steady-state goodput versus delay  $d$ .

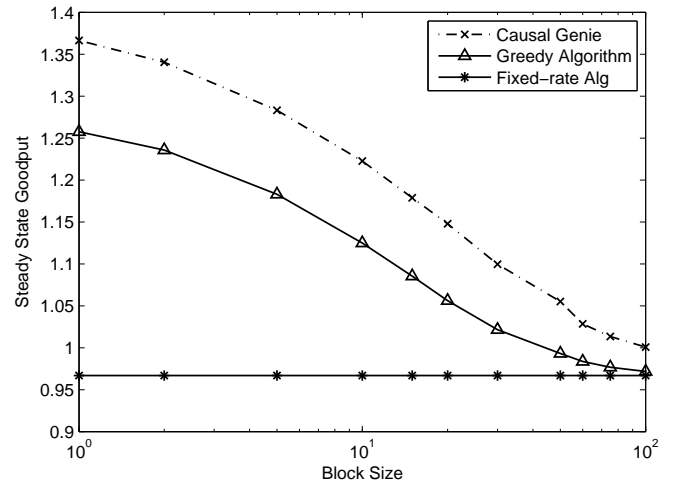


Fig. 4. Steady-state goodput versus block size  $n$ .



## V. CONCLUSION

In this paper we proposed a rate adaptation system for Rayleigh-fading Markov channels. Our system uses an algorithm that updates the constellation size iteratively, based on the received ACK packet. The algorithm first updates the conditional probability density function of the SNR given the most recent ACK packet. Then, using this function, it chooses the constellation size that maximizes the expected goodput.

We illustrated that our system can achieve a significant goodput improvement compared to a non-adaptive system, despite the very coarse CSI it uses. Indeed, we showed that the improvement over the non-adaptive algorithm with the highest possible throughput can be as high as 30%. Moreover, the goodput achieved by our system is within 8% of the highest achievable goodput with full CSI.

Also, we proposed a block-level version of our algorithm that treats the SNR and error rate as constant over a block of packets. This results in a complexity reduction that is proportional to the block length at the expense of reduced goodput. We numerically analyzed the trade-off between the goodput and the block size and showed that the goodput loss remains small for relatively small block sizes.

In the future, we plan to extend our results in order to better understand the value of link-layer feedback. For this, we plan to pursue an information theoretic analysis to compare the achieved goodput of our algorithm with the capacity of the channel for different grades of side information. We also plan to apply the ideas that we developed here to analyze the achievable goodput in OFDM systems with limited feedback.

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## APPENDIX

Here, we derive the expression for  $p(\gamma_t|\gamma_{t-d})$  given in (15). Let  $g_{t,R}$  and  $g_{t,I}$  be the real and imaginary parts of channel gain,  $g_t$ . Also let  $g_{t-d} = |g_{t-d}|e^{j\theta}$ ,  $\theta \sim U(0, 2\pi)$ . Then

$$p(\gamma_t|\gamma_{t-d}) = \int_0^{2\pi} p(\gamma_t|\gamma_{t-d}, \theta) p(\theta) d\theta \quad (16)$$

We first find  $p(|g_t|\gamma_{t-d}, \theta)$  to evaluate  $p(|g_t|\gamma_{t-d})$ . Since,

$$g_t = (1 - \alpha)^d |g_{t-d}| e^{j\theta} + Z \quad (17)$$

for  $Z = \alpha \sum_{i=0}^{d-1} (1 - \alpha)^i w_{t-i}$  and  $|g_t| = \sqrt{\frac{\gamma_t}{K}}$ , conditioned on  $\gamma_{t-d}$  and  $\theta$ ,  $g_{t,R}$  and  $g_{t,I}$  are both Gaussian with mean

$$E[g_{t,R}|\gamma_{t-d}, \theta] = (1 - \alpha)^d \sqrt{\frac{\gamma_{t-d}}{K}} \cos \theta \quad (18)$$

and

$$E[g_{t,I}|\gamma_{t-d}, \theta] = (1 - \alpha)^d \sqrt{\frac{\gamma_{t-d}}{K}} \sin \theta \quad (19)$$

respectively and variance  $\sigma_Z^2 (= E(Z^2))$ . Thus conditional on  $\gamma_{t-d}$  and  $\theta$ ,  $|g_t| = g_{t,R}^2 + g_{t,I}^2$  is Rician [9, p. 78]:

$$p(|g_t|\gamma_{t-d}, \theta) = \frac{|g_t|}{\sigma_Z^2} \exp\left(-\frac{(|g_t|^2 + (1 - \alpha)^{2d} \frac{\gamma_{t-d}}{K})}{2\sigma_Z^2}\right) \times I_0\left(\frac{|g_t|(1 - \alpha)^d \sqrt{\frac{\gamma_{t-d}}{K}}}{\sigma_Z^2}\right). \quad (20)$$

One can see that, given  $\gamma_{t-d}$ ,  $|g_t|$  is independent of  $\theta$ . Since  $\gamma_t = K|g_t|^2$ , we have

$$p(\gamma_t|\gamma_{t-d}) = \frac{1}{2K\sigma_Z^2} \exp\left(-\frac{(\frac{\gamma_t}{K} + (1 - \alpha)^{2d} \frac{\gamma_{t-d}}{K})}{2\sigma_Z^2}\right) \times I_0\left(\frac{(1 - \alpha)^d \sqrt{\gamma_t \gamma_{t-d}}}{K\sigma_Z^2}\right). \quad (21)$$

Hence combining (16) and (21), we get

$$p(\gamma_t|\gamma_{t-d}) = \frac{1}{2K\sigma_Z^2} \exp\left(-\frac{(\gamma_t + (1 - \alpha)^{2d} \gamma_{t-d})}{2K\sigma_Z^2}\right) \times I_0\left(\frac{(1 - \alpha)^d \sqrt{\gamma_t \gamma_{t-d}}}{K\sigma_Z^2}\right). \quad (22)$$

Finally, plugging  $\sigma_Z^2 = \frac{\alpha}{2-\alpha} (1 - (1 - \alpha)^{2d})$  into (22) yields (15).