Adaptive Compressive Noncoherent Change Detection

Phil Schniter

International BASP Frontiers Workshop

Villars-sur-Ollon

1/30/2013

Outline:

1. Noncoherent Change Detection
2. Compressive Noncoherent Change Detection
3. Adaptive Sensing

Change Detection:

- Given a **reference signal** (or image) $\boldsymbol{r} \in \mathbb{C}^N$ and a **signal-under-test** $\boldsymbol{x} \in \mathbb{C}^N$, how do we detect the pixels that are changed?

- Set up a model:

$$\forall n: \ x_n = s_n c_n + (1 - s_n)(r_n + d_n) \ \text{ with } \textbf{unknown...}$$

$$\begin{cases} s_n \in \{0,1\} & \text{change indicators} \\ c_n & \text{values of changed pixels} \\ d_n & \text{small variations at ``unchanged'' pixels,} \end{cases}$$

where $c_n \sim \text{i.i.d } p_C(\cdot), \ d_n \sim \text{i.i.d } p_D(\cdot), \ s_n \sim \text{i.i.d } p_S(\cdot)$

- Various optimal detectors can be formulated as a **likelihood ratio test**:

$$\mathsf{LR}(x_n, r_n) = \frac{p(x_n, r_n | s_n = 1)}{p(x_n, r_n | s_n = 0)} = \frac{p_C(x_n)}{p_D(x_n - r_n)}$$

- Intuition: look for outliers in **difference signal** $x_n - r_n$.

Noncoherent Change Detection:

- Now suppose that $\boldsymbol{r}$ and $\boldsymbol{x}$ are **phase incoherent**.
- One application is **radar image change detection in foliage**, where pixel phases can vary significantly across looks due to wind-induced motion.
- A possible model is:

$$\forall n: \;\; x_n = s_n c_n + (1 - s_n)(r_n e^{j\theta_n} + d_n)$$

where $\theta_n \sim$ i.i.d $\mathcal{U}[0, 2\pi)$ implies complete phase uncertainty.

- Change detection is still a textbook problem,

$$\text{GLR}(x_n, r_n): \frac{p_C(x_n)}{\min_{\theta_n} p_D(x_n - r_n e^{j\theta_n})} = \frac{p_C(|x_n|)}{p_D(|x_n| - |r_n|)} \text{ for circular } C \;\&\; D.$$

Intuition: look for outliers in **magnitude difference** $|x_n| - |r_n|$.

Compressive Noncoherent Change Detection:

Now consider **noisy compressive linear observations** $\boldsymbol{y} \in \mathbb{C}^M$ with $M < N$:

$$\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{w}, \quad \boldsymbol{w} \sim \mathcal{CN}(\boldsymbol{0}, \nu^w \boldsymbol{I})$$

Challenges:

- The signal $\boldsymbol{x}$ **is not directly observed**:

  $\Rightarrow$ Cannot implement standard noncoherent detection without $|x_n|$.

- The signal $\boldsymbol{x}$ is generally **non-sparse/compressible**:

  $\Rightarrow$ Cannot use standard sparse-reconstruction to recover $\boldsymbol{x}$ from $\boldsymbol{y}$.

Opportunities:

- With sparse changes, we know **most magnitudes $|x_n|$, approximately**, and thus have a strong prior on $\boldsymbol{x}$.
- In practice, the change-pattern $\boldsymbol{s}$ is not i.i.d, but **spatially clustered**.

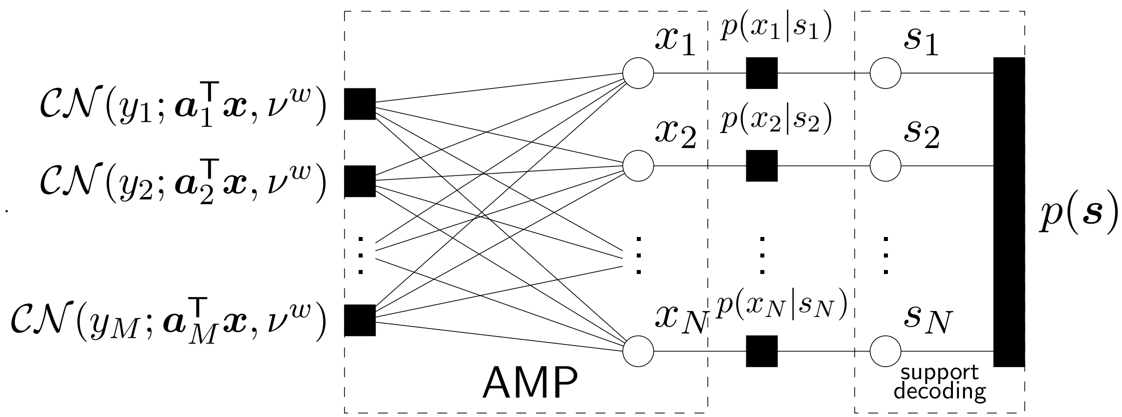
Proposed Approach:

We assume the generative mixture model

$$x_n = s_n c_n + (1 - s_n)(r_n e^{j\theta_n} + d_n) \quad \text{and} \quad \boldsymbol{y} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{w}$$

$$\begin{cases} s_n & \sim \{0,1\} \text{ Markov} \\ c_n & \sim \mathcal{CN}(0, \nu^r) \text{ i.i.d} \\ d_n & \sim \mathcal{CN}(0, \nu^d) \text{ i.i.d}, \;\; \nu^d \ll \nu^r \\ \theta_n & \sim \mathcal{U}[0, 2\pi) \text{ i.i.d} \end{cases} \qquad \Big\{ \boldsymbol{w} \sim \mathcal{CN}(\boldsymbol{0}, \nu^w \boldsymbol{I})$$

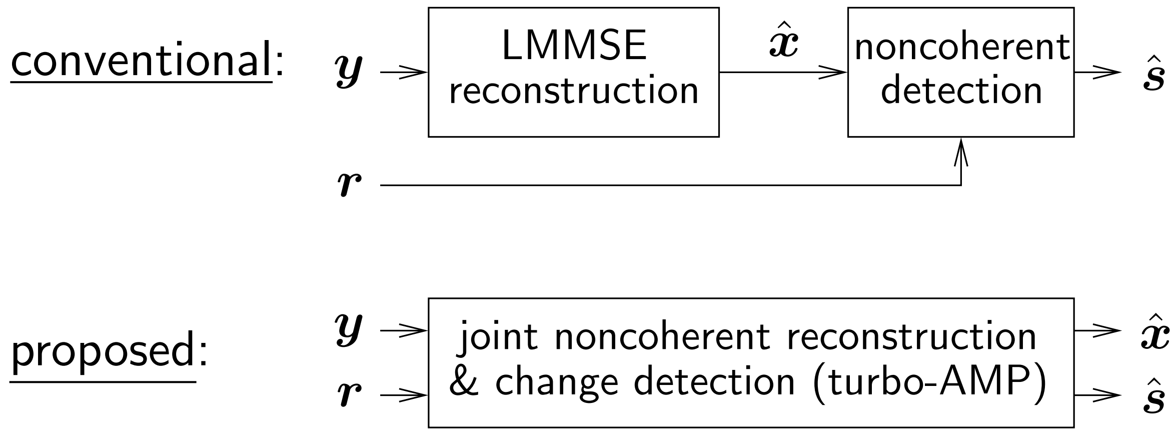
leading to the factor graph



and then perform inference via **"turbo" approximate-message-passing**.

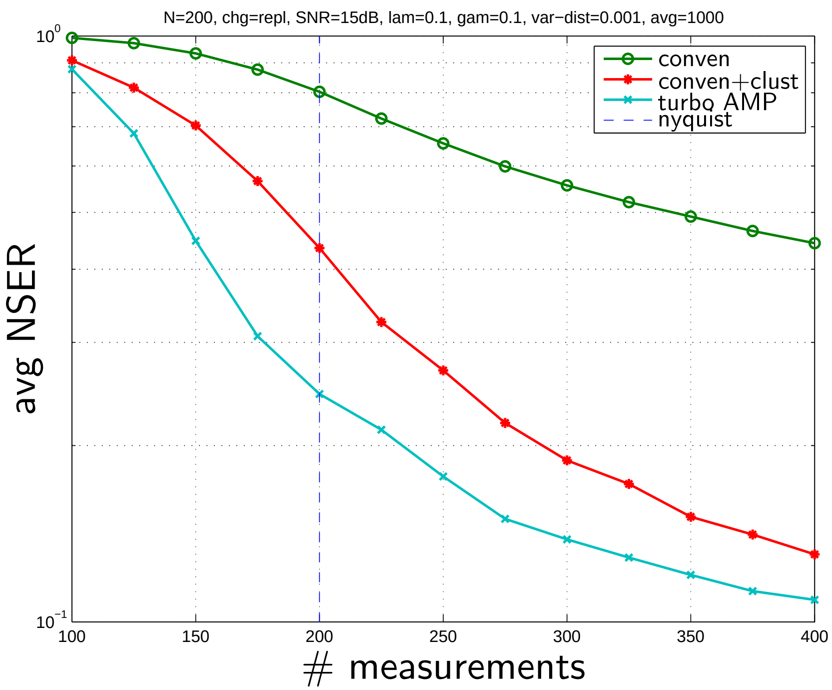
Numerical Example:

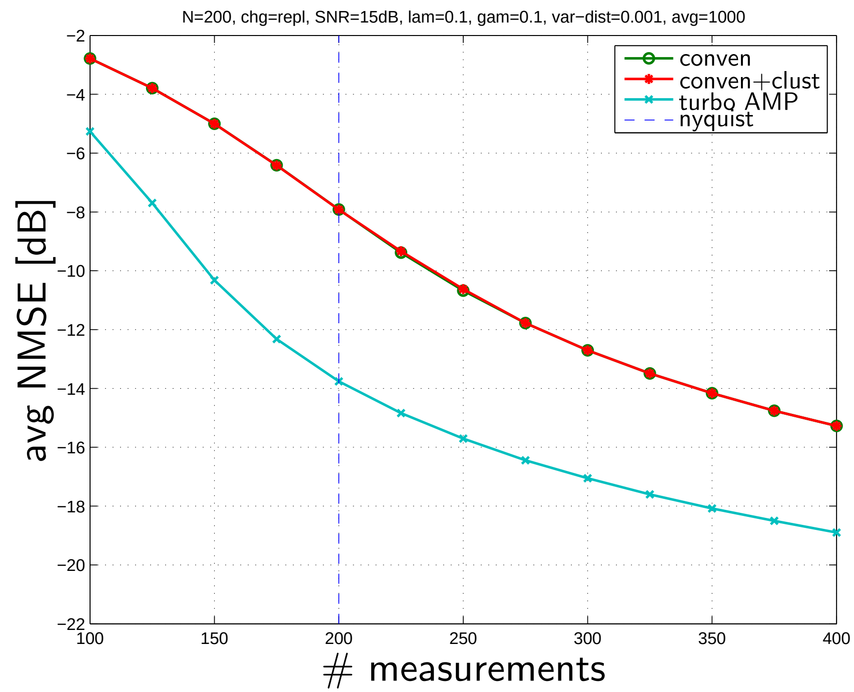
- We compare two schemes:



- Simulation parameters:
  - signal length $N = 200$,
  - changes: 1D Markov chain with rate 0.1 and avg cluster length $= 11$.
  - reference-to-disturbance ratio $\frac{\nu_r}{\nu_d} = 30$ dB,
  - signal-to-noise ratio $= 15$ dB,
  - sensing matrix: $\{A_{mn}\} \sim$ i.i.d $\mathcal{N}(0, M^{-1})$

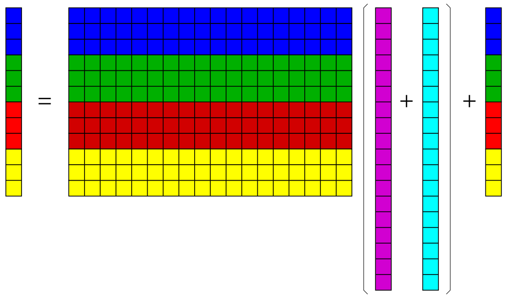
Numerical Example:





- AMP-based joint reconstruction-and-change-detection outperforms the conventional method in both NSER and NMSE, even when the conventional detector can exploit clustered changes.

Adaptive Sensing:



- Now consider the **multi-step** observation model

$$\boldsymbol{y}_t = \boldsymbol{A}_t \boldsymbol{x} + \boldsymbol{w}_t, \qquad t = 1 \ldots T$$

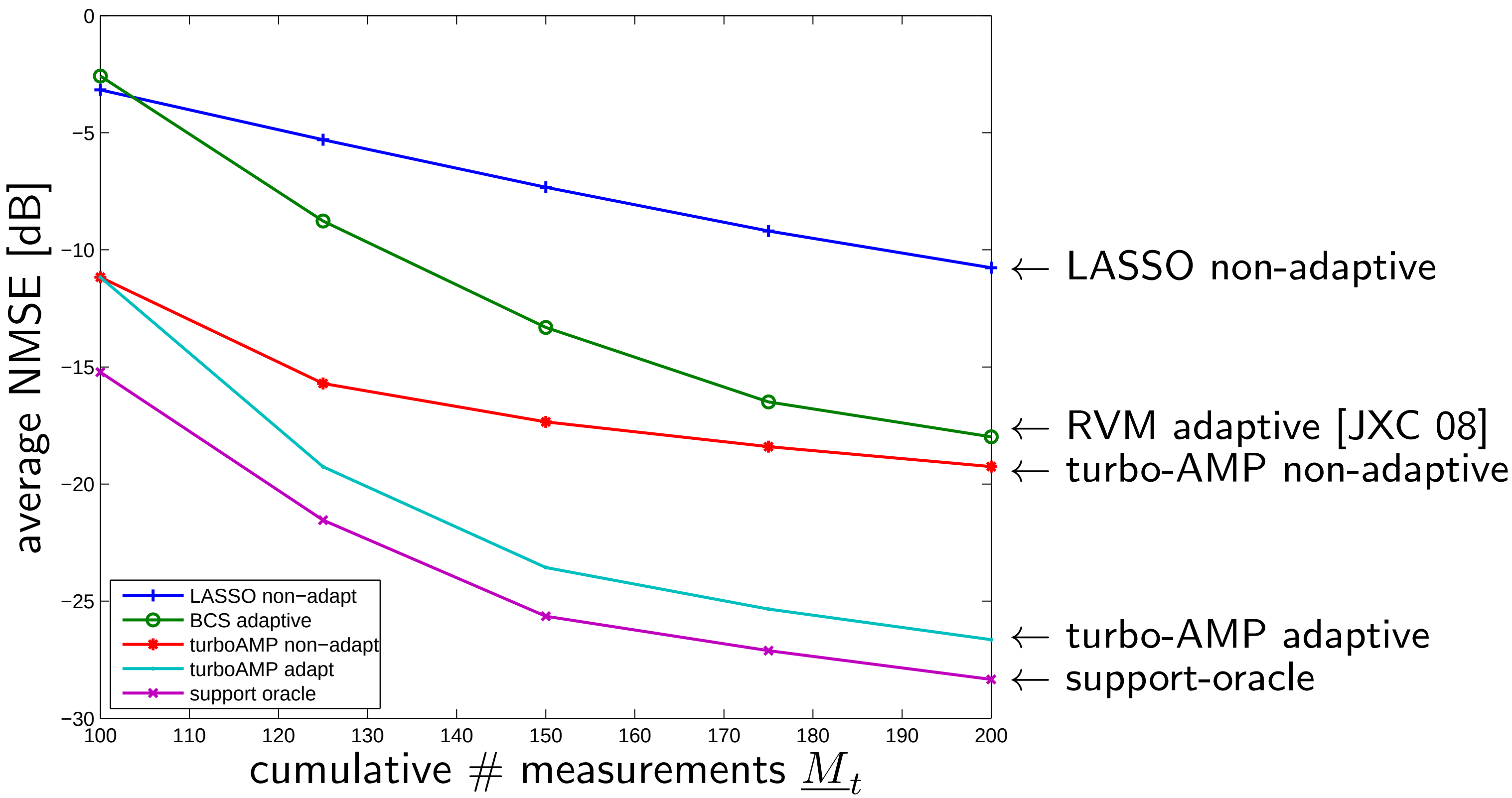
  and the **adaptation** of $\boldsymbol{A}_t$ (s.t. $\|\boldsymbol{A}_t\|_F^2 \leq \mathcal{E}$) using knowledge gained from previous measurements $\underline{\boldsymbol{y}}_{t-1} \triangleq \{\boldsymbol{y}_\tau\}_{\tau=1}^{t-1}$.

- To infer $\boldsymbol{x}$, the approach known as **Bayesian experimental design** chooses $\boldsymbol{A}_t$ to maximize the **mutual information** $\mathrm{I}(\boldsymbol{X}; \boldsymbol{Y}_t)$ between random vectors $\boldsymbol{X} \sim p(\boldsymbol{x}|\underline{\boldsymbol{y}}_{t-1})$ and $\boldsymbol{Y}_t \sim p(\boldsymbol{y}_t|\underline{\boldsymbol{y}}_{t-1}; \boldsymbol{A}_t)$.

- For **Gaussian signal and noise**, we previously established that the design of MI-maximizing $\boldsymbol{A}_t$ is a **waterfilling** problem [Schniter CAMSAP 11].

- Since **turbo-AMP** produces an accurate **Gaussian posterior approximation**, it partners well with waterfilling-based adaptation. For **structured-sparse signal recovery**, this combination has been shown to yield recovery-MSE near **oracle bounds** [Schniter CAMSAP 11].

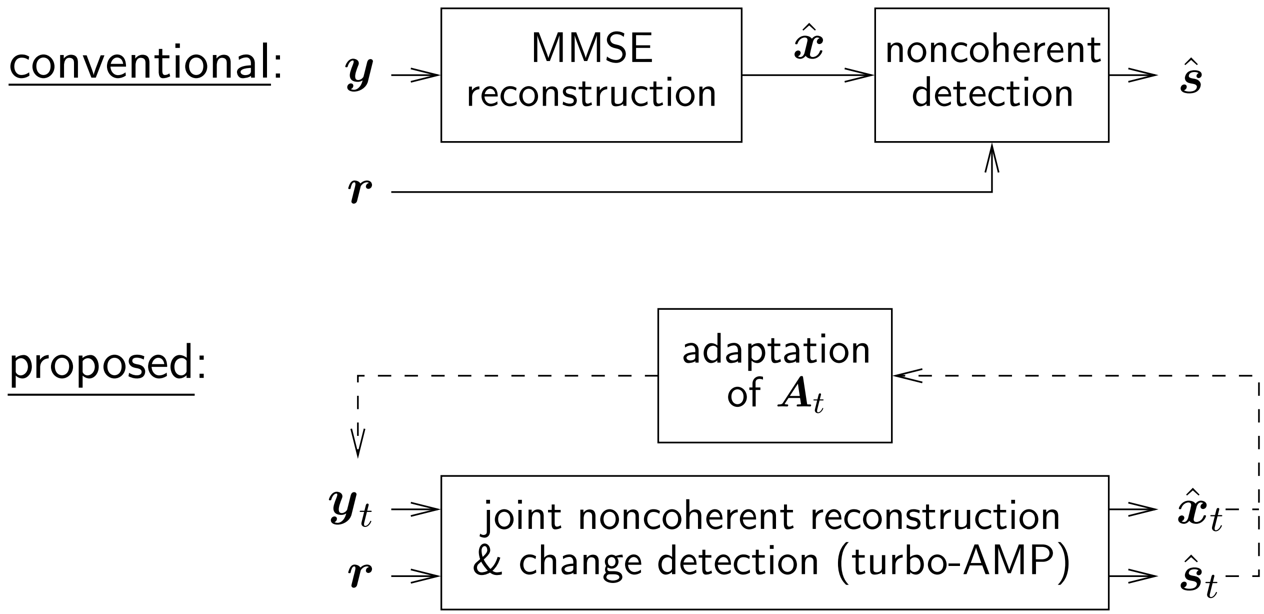
NMSE versus cumulative # measurements [Schniter CAMSAP 11]:



- Note gains from structured sparsity, adaptivity, and the combination.
- **Adaptive turbo-AMP** performs **1.5 dB from the support-oracle bound**!

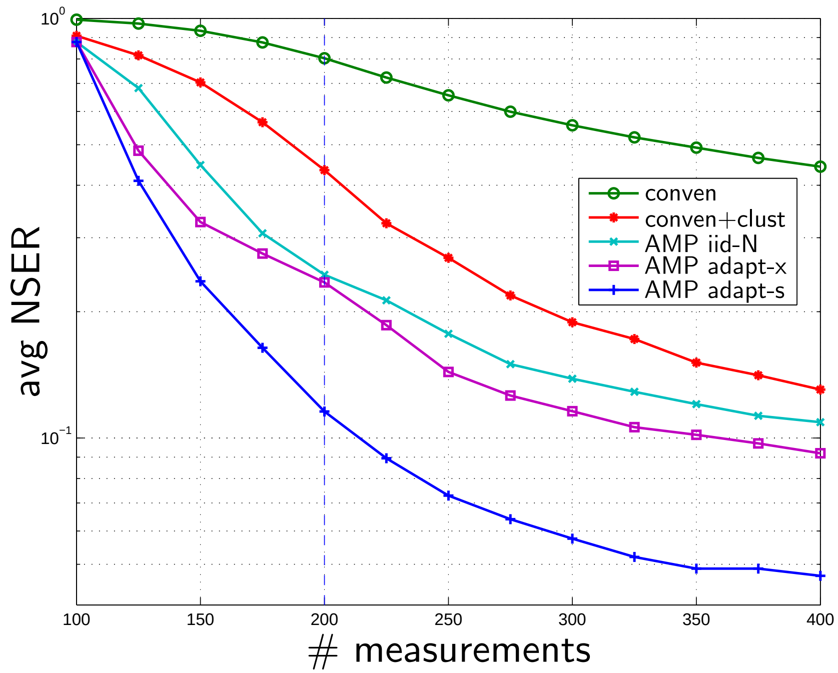
Waterfilling-based Adaptation for Noncoherent Change Detection:

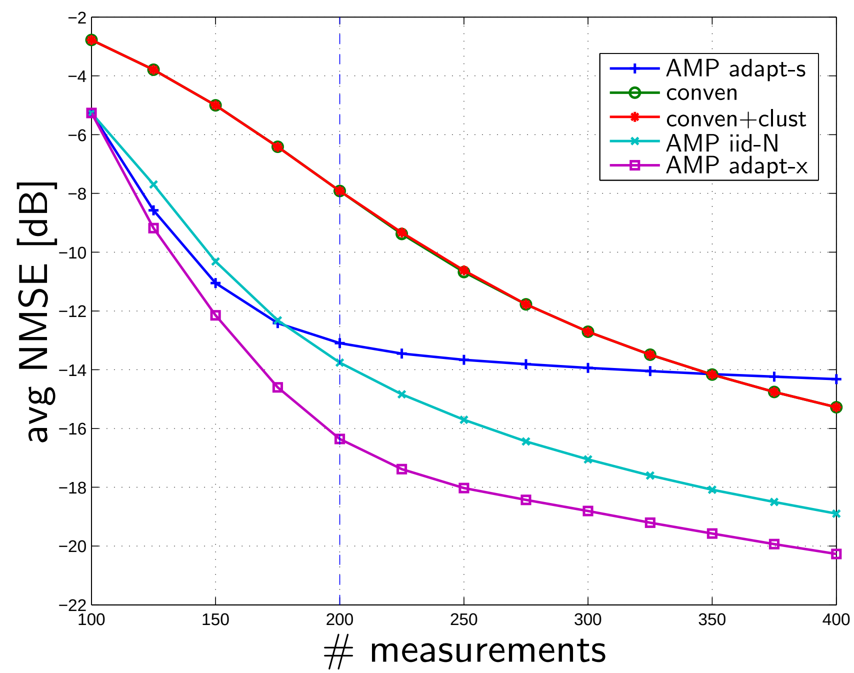
- We now add **waterfilling-based adaptive sensing** to our **noncoherent change detection** scheme.



- To minimize **signal-recovery normalized MSE** (NMSE), we perform waterfilling based on a Gaussian approximation of $p(\boldsymbol{x}|\underline{\boldsymbol{y}}_{t-1})$.
- To minimize the **normalized change-support error rate** (NSER), we perform waterfilling based on a Gaussian approximation of $p(\boldsymbol{s}|\underline{\boldsymbol{y}}_{t-1})$.

Numerical Example:





Notice that:

- the matrices designed to improve the recovery of the change pattern $\boldsymbol{s}$ do significantly improve the NSER (left), and
- those designed to improve the recovery of signal $\boldsymbol{x}$ do improve NMSE (right),
- but not vice versa!