# Fast Near-Optimal Noncoherent Decoding for Block Transmission over Doubly Dispersive Channels

Sung-Jun Hwang and Phil Schniter

Asilomar 2007

## Problem Description:

- Coded block transmission over a quickly time-varying frequency selective channel.
- Channel realizations *unknown*, but channel statistics known.
- Goal: near-optimal decoding with very low complexity and very few pilots.

## Approach:

- Turbo reception (soft noncoherent equalization $\rightleftarrows$ soft decoding).
- Soft decoder: off-the-shelf.
- Soft noncoherent equalizer: a novel design leveraging. . .
  - tree-search based on M-algorithm,
  - basis expansion model (BEM) for channel variation,
  - fast metric update (*linear* complexity).

## Channel Model:

Received samples are $\{r_n\}_{n=0}^{N-1}$, where

$$r_n = \sum_{l=0}^{N_h-1} h_{n,l} s_{n-l} + v_n$$

$h_{n,l}$ : time-$n$ response to an impulse at time $(n-l)$

$N_h$ : discrete delay spread

$\{s_n\}_{n=0}^{N-1}$ : symbols mapped from coded bits $\{x_k\}_{k=0}^{QN-1}$ into the $2^Q$-ary symbol alphabet $\mathcal{S}$

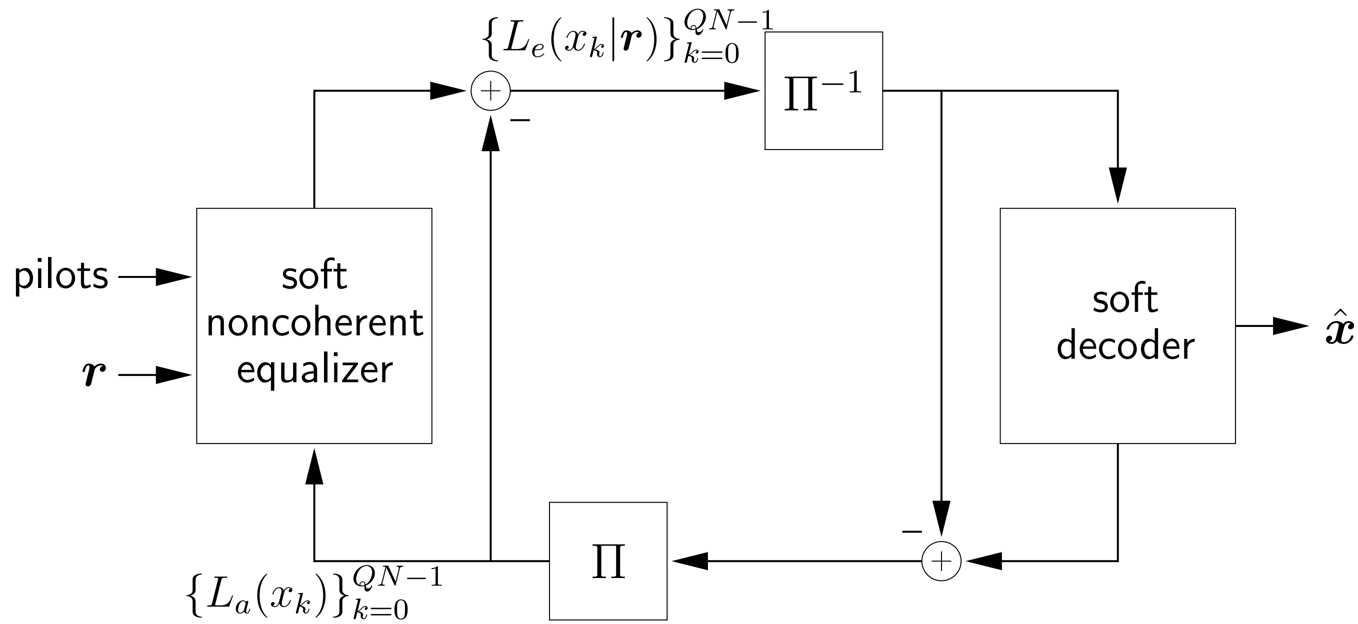
$\{v_n\}_{n=0}^{N-1}$ : CWGN with variance $\sigma^2$.

We assume WSSUS Rayleigh fading:

$$E\{h_{n,l} h^*_{n-m,l-p}\} = \rho_m \sigma_l^2 \delta_p,$$

where the autocorrelation $\{\rho_m\}$ and delay-power profile $\{\sigma_l^2\}$ are known.

# Turbo Reception:




$$L_e(x_k|\boldsymbol{r}) \;=\; \ln \frac{\sum_{\boldsymbol{x}:\, x_k=1} \exp \mu(\boldsymbol{x})}{\sum_{\boldsymbol{x}:\, x_k=0} \exp \mu(\boldsymbol{x})} - L_a(x_k) \qquad \text{"extrinsic LLR"}$$

$$\mu(\boldsymbol{x}) \;=\; \ln p\big(\boldsymbol{r}|\boldsymbol{s}(\boldsymbol{x})\big) + \sum_{i:\, x_i=1} L_a(x_i) \qquad \text{"MAP metric"}$$

Need $\mathcal{O}(2^{QN})$ evaluations of $\mu(\boldsymbol{x})$ $\rightsquigarrow$ *Computationally infeasible*!!

# Simplified LLR Evaluation:

The "max-log" approximation:

$$L_e(x_k|\boldsymbol{r}) \approx \max_{\boldsymbol{x}\in\mathcal{L}\cap\{\boldsymbol{x}:x_k=1\}} \mu(\boldsymbol{x}) - \max_{\boldsymbol{x}\in\mathcal{L}\cap\{\boldsymbol{x}:x_k=0\}} \mu(\boldsymbol{x}) - L_a(x_k)$$

$$\mathcal{L} \ : \ \text{set containing the } M \text{ most probable } \boldsymbol{x},$$

requires only a few evaluations of $\mu(\boldsymbol{x})$. But how complex is this?

Say $\boxed{\boldsymbol{r} = \boldsymbol{S}\boldsymbol{h} + \boldsymbol{v}}$ where $\boldsymbol{h} \sim \mathcal{CN}(\boldsymbol{0}, \boldsymbol{R}_h)$ and $\boldsymbol{v} \sim \mathcal{CN}(\boldsymbol{0}, \sigma^2\boldsymbol{I})$. Then

$$\boldsymbol{r}|\boldsymbol{s} \sim \mathcal{CN}(\boldsymbol{0}, \underbrace{\boldsymbol{S}\boldsymbol{R}_h\boldsymbol{S}^H + \sigma^2\boldsymbol{I}}_{\boldsymbol{\Phi}(\boldsymbol{s})})$$

$$\Rightarrow \quad \ln p(\boldsymbol{r}|\boldsymbol{s}) = -\boldsymbol{r}^H\boldsymbol{\Phi}^{-1}\boldsymbol{r} - \ln\det\boldsymbol{\Phi} - N\ln\pi$$

$$\Rightarrow \quad \mu(\boldsymbol{x}) = -\boldsymbol{r}^H\boldsymbol{\Phi}^{-1}\boldsymbol{r} - \ln\det\boldsymbol{\Phi} - N\ln\pi + \sum_{i:\, x_i=1} L_a(x_i)$$

$\Rightarrow$ direct evaluation of $\mu(\boldsymbol{x})$ is $\mathcal{O}(N^3)$. *Still quite expensive!*

## Fast Soft Noncoherent Equalization:

We propose a novel equalization algorithm, based on

- efficient tree search to find best $M$ bit sequences $\mathcal{L}$, and
- fast recursive update of MAP metric $\mu(\boldsymbol{x})$ using
  - a basis expansion model (BEM) for the channel's time-variation,
  - recursive update of the implicit MMSE channel estimate $\hat{\boldsymbol{\theta}}$.

The result is *near-MAP* performance with complexity that is

- *linear* in the block length, and
- *quadratic* in the channel length.

# BEM Approximation:

$$h_{n,l} \approx \sum_{p=0}^{N_b-1} b_{n,p}\theta_{p,l} \quad \text{for } n \in \{0,\dots,N-1\}.$$

$$\begin{aligned} \{b_{n,p}\}_{n=0}^{N-1} &: p^{th} \text{ basis waveform} \\ N_b &: \text{number of basis waveforms} \\ \theta_{p,l} &: \text{coefficient for } p^{th} \text{ basis waveform and } l^{th} \text{ lag.} \end{aligned}$$

Basis choices include:

- complex exponential basis: $b_{n,p} = e^{j\frac{2\pi}{N}(p-\frac{N_b-1}{2})n}$,
- polynomial basis: $b_{n,p} = n^p$
- Karhunen-Loeve basis: $\{b_{n,p}\}_{n=0}^{N-1}$ is the $p^{th}$ largest eigenvector of the Toeplitz matrix defined from the autocorrelation $\{\rho_m\}_{m=0}^{N-1}$.

Note: $N_b = N$ yields zero approximation error, though typically $N_b = 2$.

# Sequential Processing based on the BEM:

$$\boxed{\boldsymbol{r}_n \;:=\; [r_0, r_1, \ldots, r_n]^T \;=\; \boldsymbol{B}_n \boldsymbol{S}_0^n \boldsymbol{\theta} + \boldsymbol{v}_n}$$

where, by example,

$$\underbrace{\begin{bmatrix} r_0 \\ r_1 \\ r_2 \end{bmatrix}}_{\boldsymbol{r}_2} = \underbrace{\begin{bmatrix} \boldsymbol{b}_0^H & & \\ & \boldsymbol{b}_1^H & \\ & & \boldsymbol{b}_2^H \end{bmatrix}}_{\boldsymbol{B}_2} \underbrace{\begin{bmatrix} s_0 \boldsymbol{I}_{N_b} & s_{-1} \boldsymbol{I}_{N_b} \\ s_1 \boldsymbol{I}_{N_b} & s_0 \boldsymbol{I}_{N_b} \\ s_2 \boldsymbol{I}_{N_b} & s_1 \boldsymbol{I}_{N_b} \end{bmatrix}}_{\boldsymbol{S}_0^2} \underbrace{\begin{bmatrix} \boldsymbol{\theta}_0 \\ \boldsymbol{\theta}_1 \end{bmatrix}}_{\boldsymbol{\theta}} + \underbrace{\begin{bmatrix} v_0 \\ v_1 \\ v_2 \end{bmatrix}}_{\boldsymbol{v}_2}$$

$$\begin{aligned} \boldsymbol{b}_n &= [b_{n,0}, \ldots, b_{n,N_b-1}]^H && : \text{time-}n \text{ basis elements} \\ \boldsymbol{\theta}_l &= [\theta_{0,l}, \ldots, \theta_{N_b,l}]^T && : \text{lag-}l \text{ BEM coefficients.} \end{aligned}$$

Note:

- $\boldsymbol{\theta} \in \mathbb{C}^{N_b N_h}$ contains all unknown channel coefficients,
- $\boldsymbol{S}_0^n$ contains data symbols $\boldsymbol{s}_n := [s_0, s_1, \ldots, s_n]^T$.

## Fast Metric Update:

Say $\boldsymbol{x}_n$ contains bits from $\boldsymbol{s}_n$. Can compute $\mu(\boldsymbol{x}_n)$ given $\mu(\boldsymbol{x}_{n-1})$ via

$$
\begin{aligned}
\boldsymbol{a}_n &= [s_n \boldsymbol{b}_n^H, s_{n-1}\boldsymbol{b}_n^H, \cdots, s_{n-N_h+1}\boldsymbol{b}_n^H]^H \in \mathbb{C}^{N_h N_b} \\
\boldsymbol{d}_n &= \boldsymbol{\Sigma}_{n-1}^{-1}\boldsymbol{a}_n \\
\alpha_n &= (1 + \boldsymbol{a}_n^H \boldsymbol{d}_n)^{-1} \\
\boldsymbol{\Sigma}_n^{-1} &= \boldsymbol{\Sigma}_{n-1}^{-1} - \alpha_n \boldsymbol{d}_n \boldsymbol{d}_n^H \\
\mu(\boldsymbol{x}_n) &= \mu(\boldsymbol{x}_{n-1}) - \frac{\alpha_n}{\sigma^2}|r_n - \boldsymbol{a}_n^H \hat{\boldsymbol{\theta}}_{n-1}| - \ln(\pi\alpha_n) + \sum_{i:\, x_i=1, x_i\in s_n} L_a(x_i) \\
\hat{\boldsymbol{\theta}}_n &= (\boldsymbol{I}_{N_h N_b} - \alpha_n \boldsymbol{d}_n \boldsymbol{a}_n^H)\hat{\boldsymbol{\theta}}_{n-1} + (1 - \alpha_n \boldsymbol{d}_n^H \boldsymbol{a}_n) r_n \boldsymbol{d}_n,
\end{aligned}
$$

using only $\boxed{2(N_h N_b)^2 + 9N_h N_b + 8}$ multiplications!

# Fast Tree Search:

Breadth-first search via the M-algorithm:

- Say $\mathcal{L}'_n$ contains the $M$ "best" estimates of $\boldsymbol{x}_n$. For each extension $\boldsymbol{x}_{n+1} = \left[\begin{smallmatrix} x \\ \boldsymbol{x}_n \end{smallmatrix}\right]$, where $\boldsymbol{x}_n \in \mathcal{L}'_n$ and $x \in \{0,1\}^Q$, calculate the metric $\mu(\boldsymbol{x}_{n+1})$. Then collect the $M$ best extensions in the set $\mathcal{L}'_{n+1}$.
- Doing this for $n = 0, \ldots, N-1$ requires the evaluation of $M2^QN$ MAP metrics, yielding $\mathcal{L}' := \mathcal{L}'_{N-1}$, an estimate of the $M$ most probable bit vectors.
- Performance almost indistinguishable from full search (i.e., $\mathcal{L}' \approx \mathcal{L}$).

In total, $\boxed{2M2^QN(N_bN_h)^2}$ multiplications are required to compute the MAP metrics $\{\mu(\boldsymbol{x}) : \boldsymbol{x} \in \mathcal{L}'\}$. Note that this complexity is

- *linear* in the block length $N$,
- *quadratic* in the channel length $N_bN_h$.

## Construction of the Transmission Block:

Pilots:

- One pilot symbol sufficient to resolve channel/data phase ambiguity.
- $N_p > 1$ pilots provide a good "initialization" of $\mu(\boldsymbol{x}_{N_p})$, helping improve the accuracy of tree search.
- $N_p \geq N_h N_b$ needed for channel-estimation followed by coherent decoding.

Guard:

- A ZP guard interval of length $N_h - 1$ prevents inter-block interference and enables capture of diversity from delay spread.
- Note: we hope to capture Doppler diversity through iteration with soft decoder.

# Numerical Experiments (Setup):

Transmitter:

- rate-$\frac{1}{2}$ LDPC coding, frame length $4096$, QPSK ($Q = 2$),
- block length $N = 64$,
- $N_p \in \{3, 6, 9\}$ pilots.

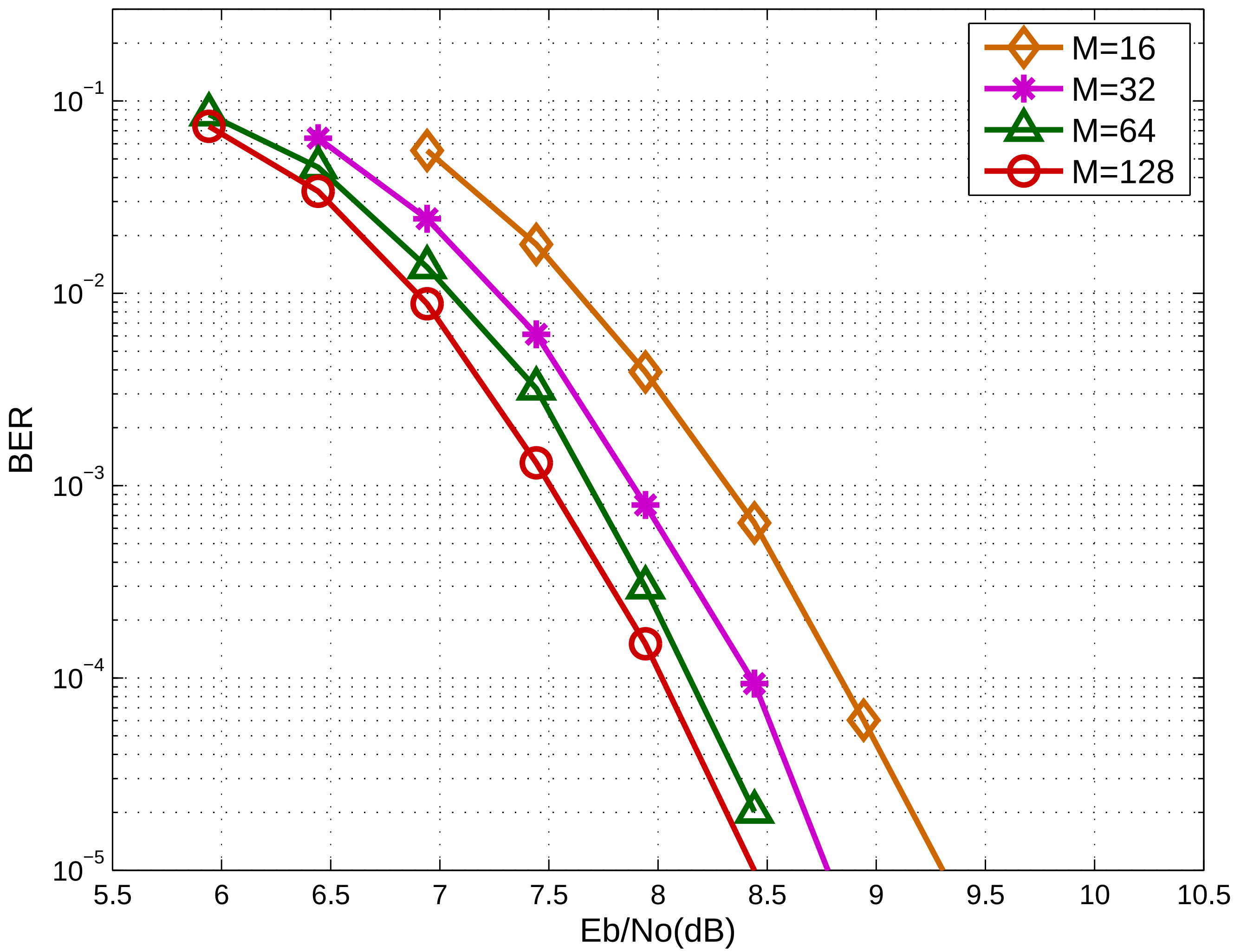
Channel:

- WSSUS Rayleigh (via Jakes) with $N_h = 3$ taps at $f_{\mathsf{D}} T_s = 0.002$.

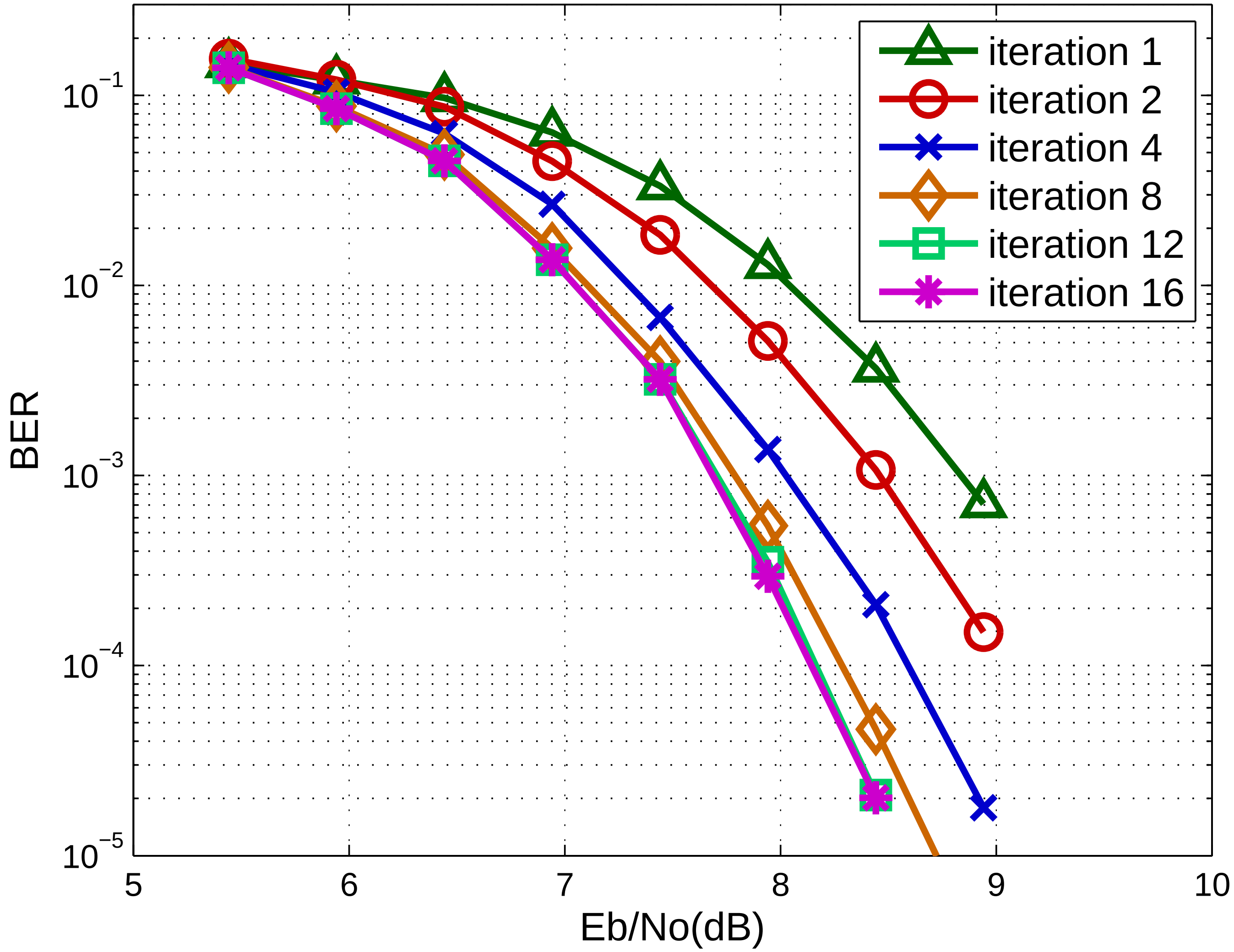
Receiver:

- KL-basis with $N_b = 2$,
- search parameter $M \in \{16, 32, 64, 128\}$,
- inner (i.e., LDPC decoding) iterations $\leq 60$,
- outer (i.e., turbo) iterations $\in \{1, 2, 4, 8, 12, 16\}$.

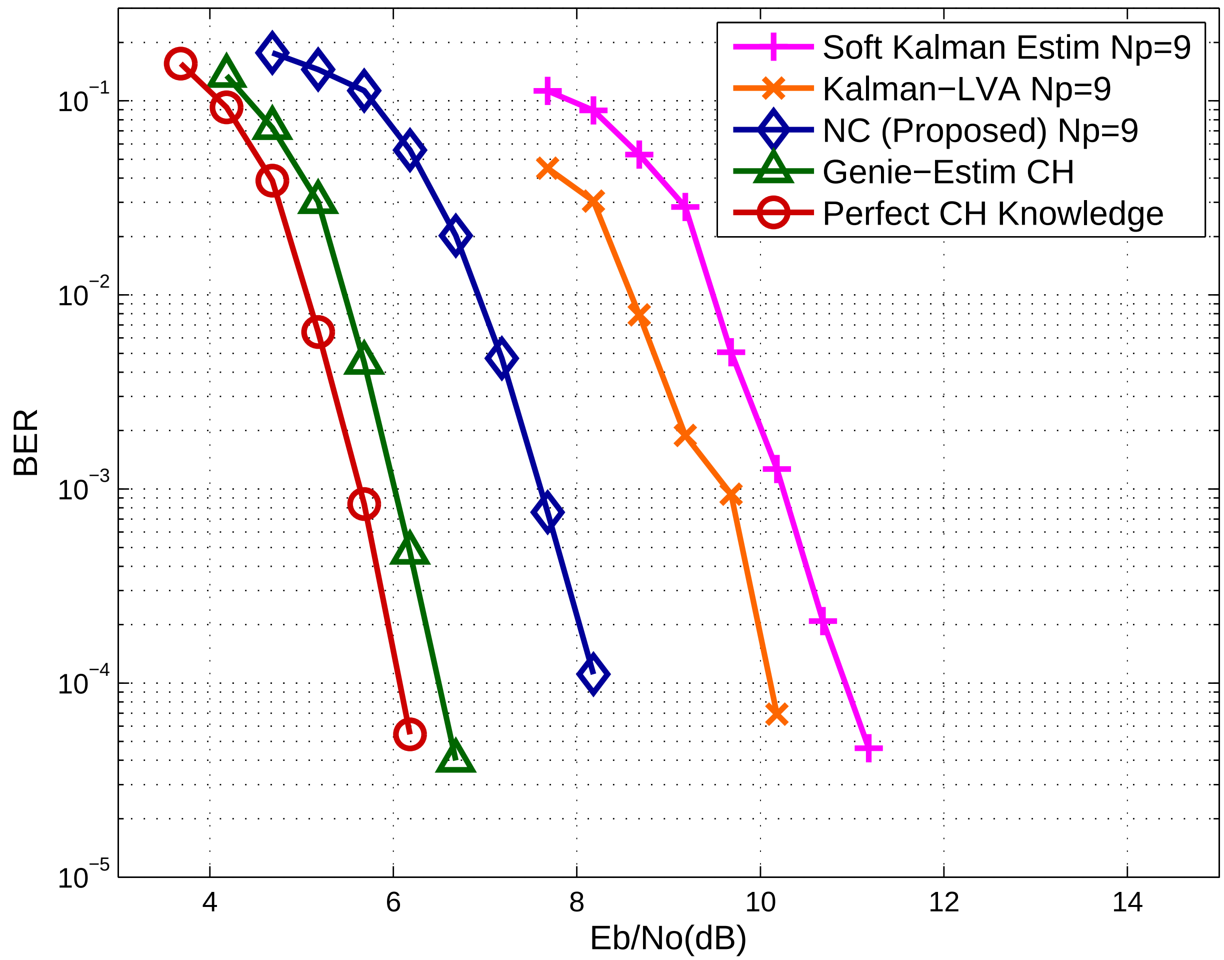
## Choice of search parameter $M$:



# Choice of # outer iterations:



# Choice of # pilots $N_p$ / genie-aided comparison:



## Numerical Experiments (Interpretations):

Genie-aided references:

1. perfectly known channel,
2. channel estimated from 100% training.

*Only about 2 dB away!*

Maximum diversity order offered by channel:

- $f_{\mathsf{D}} T_s = 0.002 \quad \Rightarrow \quad$ coherence time $= 500$ symbols.
- $4096$-symbol frame $\quad \Rightarrow \quad 8$ coherence intervals.
- $3$ taps $\times 8$ coherence intervals $=$ **24 degrees of freedom**.

*The BER slopes confirm that our scheme achieves maximum diversity!*

## Conclusions:

- We presented a novel scheme for the reception of coded transmissions over quickly varying multipath channels
- Leveraged a BEM approximation of channel, the M-algorithm, a fast recursive update for the MAP metric, and the turbo principle.
- Achieved performance $\approx 2$ dB away from genie-aided bounds at a complexity of $\boxed{\approx 2M2^Q(N_b N_h)^2}$ mults per QAM symbol.
- Only need one pilot per block, though a few more help performance.