Minimum-Entropy Blind Acquisition/Equalization for Uplink DS-CDMA

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Abstract: In this paper, we consider blind estimation of linear chip-spaced receivers for the demodulation of a particular short-code DS-CDMA mobile user under multipath propagation and in the absence of timing information. We propose a family of schemes for blind acquisition and equalization based on Donoho's Minimum Entropy principle and propose a specific algorithm that uses the second- and fourth-order moments of a pre-whitened chip-rate received signal. The proposed algorithm can be considered a near-far resistant initialization procedure for, and application of, the Constant Modulus Algorithm (CMA) to DS-CDMA.

1 Introduction

Direct sequence code division multiple access (DS-CDMA) systems have received considerable attention as a flexible means of communication between multiple mobile users and centralized base stations. Since CDMA users share the same time and frequency resources, demodulation of a particular user is principally concerned with suppression of interference from other users. The link from base station to mobiles is referred to as the "downlink" and is typically characterized by synchronous equal-power data transmission to all users. More challenging for demodulation is the "uplink," from mobiles to base station, where different user transmissions are typically asynchronous and of widely disparate power levels. Reliable demodulation might also require mitigation of multipath interference, especially in recently proposed "wideband" CDMA schemes (e.g., IMT-2000) where multipath effects can be significant.

In this paper, we consider blind estimation of linear receivers for demodulation of a (possibly weak) asynchronous CDMA user under multipath propagation. By "blind," we mean that all interfering code vectors, all user timings, and all multipath channels are unknown, and that no training information is available. Thus, obtaining reliable symbol estimates involves both blind equalization *and* blind timing acquisition of the desired user. Previously proposed schemes rely solely on the second-order statistics (SOS) of the received data and are based on either subspace-constrained optimization [1, 2] or subspace decomposition [3]. The use of constraints implies that the minimizers of such criteria are *not*, in general, global MSE minimizers. Proponents claim that "this price is paid in exchange for the advantage of not having a training sequence" [2, p. 107], but must we really pay this price? Similarly, subspace techniques assume that the signal and noise subspaces can be accurately estimated, but is this usually possible?

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In this paper, we propose a family of blind acquisition/equalization schemes that make inherent use of the higher-order statistics (HOS) of the received signal. The central concept is rooted in Donoho's Minimum Entropy (ME) principle [4], a fundamentally different approach to blind acquisition/equalization than [1, 2, 3]. A major advantage of the minimum entropy techniques is that they are capable of achieving *global* MMSE solutions under ideal conditions (to be discussed) while exhibiting robust performance in realistic (non-ideal) conditions. We present a specific ME algorithm that uses the second- and fourth-order moments of a whitened chip-rate version of the received signal. It can be considered a near-far resistant application of CMA to uplink DS-CDMA in the absence of timing information.

Notation: We use bold lowercase for vectors, bold uppercase for matrices, $(\cdot)^t$ for transposition, $(\cdot)^*$ for conjugation, and $(\cdot)^H$ for conjugate transposition. All vector/matrix indices start with zero (i.e., we denote the first element of $\mathbf{x}$ by x_0 rather than x_1), and n used as a symbol-rate index, i as a chip-rate index, k as a user index, and ν as a hypothesis index. Finally, $\lceil\cdot\rceil$ denotes ceiling, $\langle\cdot\rangle_N$ modulo-N, $\mathrm{E}\{\cdot\}$ expectation, $*$ convolution, $\|\mathbf{x}\|_p := \sqrt[p]{\sum_m |x_m|^p}$ the ℓ_p norm of $\mathbf{x}$, and $\mathbf{e}_m$ a vector with a one in the m^{th} position and zeros elsewhere.

2 DS-CDMA System Model

We formulate the uplink DS-CDMA model as follows. K mobile users transmit simultaneously to a base station, where the k^{th} user transmits a symbol (i.e., "bit") sequence $\{s_n^{(k)}\}$ indexed by n and with symbol spacing of T seconds. The users are each assigned a "short" spreading code composed of N chips with chip duration $T_c = T/N$. We shall represent the k^{th} user's code by the vector $\mathbf{c}^{(k)} := (c_0^{(k)}, \ldots, c_{N-1}^{(k)})^t$. A given user's short code is multiplied by each of his symbols prior to transmission, so that the transmitted data sequence is spaced at the chip rate.

Now consider the baseband model illustrated in Figure 1. Each mobile transmits his chip-rate sequence through a pulse shaping filter with impulse response $p(t)$. In typical applications, $p(t)$ is bandlimited to approximately $1/T_c$ using, e.g., a square-root raised-cosine filter. Before reaching the receiver, each transmitted waveform passes through a user-specific atmospheric propagation channel which includes, e.g., multipath and path-loss effects. We restrict our focus to a linear time-invariant approximation to the propagation channel and denote the resulting impulse response by $g^{(k)}(t)$. In addition to linear distortion, the propagation channel contributes additive noise $n(t)$.

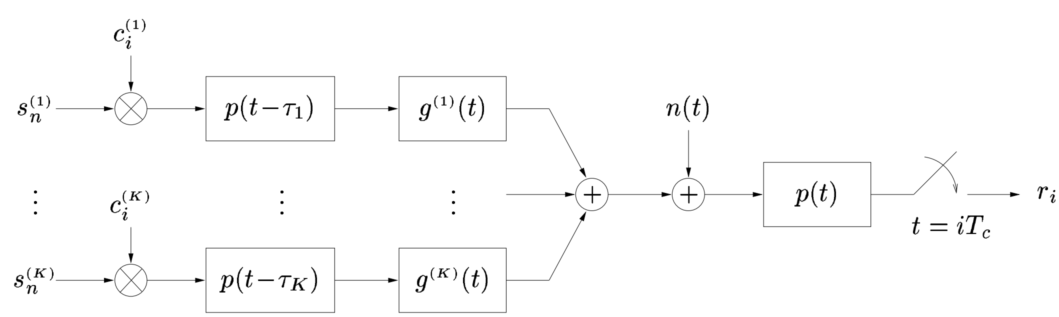

Figure 1: DS-CDMA system model with chip-sampling receiver.

2.1 Discrete Model

The receiver sees the sum of all transmitted signals plus noise and samples the output of a single "chip-matched" filter at the chip rate $1/T_c$. As is typical, we assume that the chip-matched filter is identical to the transmit filter $p(t)$. User-specific time delays $\{\tau_k\}$ are used to capture the lack of timing coordination, or asynchronicity, among users. Without loss of generality[1], time delay is assumed in the range $0 \le \tau_k < T$.

The model of Figure 1 admits the discrete equivalent of Figure 2, where $\uparrow N$ denotes upsampling and $\overleftarrow{\mathbf{c}}^{(k)} := (c^{(k)}_{N-1}, \ldots, c^{(k)}_0)^t$ the reverse-ordered code vector, and where $\mathbf{h}^{(k)} := (h^{(k)}_0, \ldots, h^{(k)}_{L_h-1})^t$ contains the T_c-spaced impulse response samples of the convolved transmit filter, propagation channel, and receiver filter:

$$h_i^{(k)} \;=\; p(t-\tau_k) * g^{(k)}(t) * p(t)\big|_{t=iT_c}. \tag{1}$$

In (1) we assumed that $p(t)$ and $g^{(k)}(t)$ are causal and time-limited so that, for all k, $p(t-\tau_k) * g^{(k)}(t) * p(t)|_{t=iT_c} = 0$ for $i \ge L_h$ and $i < 0$. Note that using one number, L_h, to characterize the length of every user's discrete channel implies that some of the $\mathbf{h}^{(k)}$ may be zero-padded. Finally, note that the sampled noise process $\{w_i\}$ in Figure 2 is given by $w_i = n(t) * p(t)|_{t=iT_c}$.

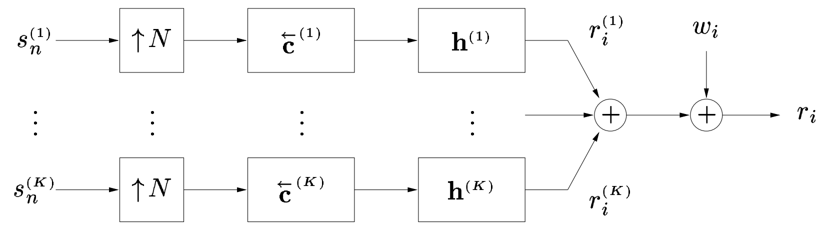

Figure 2: Discrete-time equivalent DS-CDMA system model with chip-sampling receiver.

From now on we restrict our attention to finite-length linear receivers. Collecting, say, the L_r most recent samples at time n in the vector $\mathbf{r}(n)$, it is possible to express $\mathbf{r}(n)$ as a linear combination of transmitted symbols. To see this, we first write $\mathbf{r}(n)$ in terms of specific user contributions plus noise:

$$\mathbf{r}(n) \;=\; \sum_{k=1}^{K} \mathbf{r}^{(k)}(n) + \mathbf{w}(n), \tag{2}$$

where $\mathbf{r}^{(k)}(n)$ is a vector composed of the L_r most recent noiseless chip-rate k^{th} user contributions $r_i^{(k)}$, and $\mathbf{w}(n)$ is composed of the L_r most recent chip-rate noise samples w_i (see Figure 2). The k^{th} user contribution can be written as a matrix-vector product:

$$\mathbf{r}^{(k)}(n) \;=\; \mathbf{H}^{(k)}\mathbf{C}^{(k)}\mathbf{s}^{(k)}(n), \tag{3}$$

where $\mathbf{H}^{(k)}$ is an $L_r \times L_r + L_h - 1$ Toeplitz matrix and $\mathbf{C}^{(k)}$ is a $L_r + L_h - 1 \times \lceil \frac{L_r + L_h - 1}{N} \rceil$ matrix structured as below,

$$\mathbf{H}^{(k)} := \begin{pmatrix} h_0^{(k)} & h_1^{(k)} & \cdots & h_{L_h-1}^{(k)} & & & \\ & h_0^{(k)} & h_1^{(k)} & \cdots & h_{L_h-1}^{(k)} & & \\ & & \ddots & & & \ddots & \\ & & & h_0^{(k)} & h_1^{(k)} & \cdots & h_{L_h-1}^{(k)} \end{pmatrix}, \mathbf{C}^{(k)} := \begin{pmatrix} \mathbf{c}^{(k)} & & & \\ & \mathbf{c}^{(k)} & & \\ & & \ddots & \\ & & & \mathbf{c}^{(k)} \end{pmatrix},$$

[1]Assuming stationary source processes, the received signal will be cyclo-stationary with period T. Thus, the statistics of the received sequence will be insensitive to shifts of nT in user delay.

but with the rightmost code vector possibly truncated to length $\langle L_r+L_h-2\rangle_N+1$. The vector of the k^{th} user's source symbols influencing $\mathbf{r}(n)$, namely $\mathbf{s}^{(k)}(n) := (s_n^{(k)}, \ldots, s_{n-L_s+1}^{(k)})^t$, is of length $L_s := \lceil \frac{L_r+L_h-1}{N} \rceil$. Equation (2) can then be written

$$\mathbf{r}(n) \;=\; \sum_{k=1}^{K} \mathbf{H}^{(k)}\mathbf{C}^{(k)}\mathbf{s}^{(k)}(n) + \mathbf{w}(n) \;=\; \mathcal{H}\,\mathbf{s}(n) + \mathbf{w}(n), \tag{4}$$

where the $L_r \times KL_s$ multiuser code-channel matrix $\mathcal{H}$ and the $KL_s \times 1$ multiuser source vector $\mathbf{s}(n)$ are defined by

$$\mathcal{H} \;:=\; \left(\mathbf{H}^{(1)}\mathbf{C}^{(1)}, \mathbf{H}^{(2)}\mathbf{C}^{(2)}, \ldots \mathbf{H}^{(K)}\mathbf{C}^{(K)}\right), \tag{5}$$

$$\mathbf{s}(n) \;:=\; \left(\mathbf{s}^{(1)^t}(n), \mathbf{s}^{(2)^t}(n), \ldots, \mathbf{s}^{(K)^t}(n)\right)^t. \tag{6}$$

2.2 Rank of Code-Channel Matrix $\mathcal{H}$

Expression (4) for the received DS-CDMA signal has the same basic structure seen in many multichannel deconvolution problems (e.g., wideband beamforming, wideband source separation, crosstalk suppression, and cross-polarization). The principle differences among these problems concern the properties of the "channel matrix" $\mathcal{H}$. A critical assumption typically made in analysis of the previously mentioned applications is that $\mathcal{H}$ is full column rank. Satisfaction of this condition allows, for example, perfect recovery of any source element in $\mathbf{s}(n)$ via linear processing[2] of $\mathbf{r}(n)$ in the absence of noise. Full column rank $\mathcal{H}$ and the absence of noise also makes possible the "blind" determination of these perfect-recovery linear filters using algorithms based on second- and/or higher-order statistics. Non full-column rank $\mathcal{H}$ can degrade the performance of blind algorithms, in some cases catastrophically. These reasons motivate a closer look at the rank of $\mathcal{H}$ for the specific case of asynchronous DS-CDMA under multipath propagation.

Let us consider the dimensional requirements necessary for full column rank $\mathcal{H}$. To have at least as many rows as columns, one requires $L_r \geq K\lceil \frac{L_r+L_h-1}{N} \rceil$. In the case that $K = N$, the requirement becomes $L_r \geq L_r+L_h-1+C$, where C is an integer in the range $0 \leq C < N$. It follows directly that a necessary condition for full column rank $\mathcal{H}$ with $K = N$ users is $L_h = 1$, or, in other words, the absence of asynchronism or multipath! In the case $K < N$, it can be shown that choosing $L_r > \lceil \frac{K(N+L_h-1)-N}{N-K} \rceil$ guarantees that the dimensional requirement is satisfied. To summarize, for a single-sensor DS-CDMA system with $\geq N$ users, asynchronism or multipath will ensure that $\mathcal{H}$ is *not* full column rank for any finite observation length L_r.

Practical cellular communication systems are based on the use of multiple cells, each cell composed of one base station and $\leq N$ mobiles. One of the principle advantages of CDMA in the cellular context is its potential for universal frequency reuse. This implies, however, that other-cell signals will interfere with in-cell communication. In fact, it has been suggested that the typical out-of-cell interference level is roughly half of the in-cell interference level [5]. Thus, a multi-cell model should incorporate $K \gg N$ significant users and, by the arguments above, will result in non full-column-rank $\mathcal{H}$. We note that the rank of $\mathcal{H}$ may be increased through the use of a receiver with multiple sensors [3], but the number required to achieve full rank may be prohibitively large.

[2] Perfect linear recovery follows directly from the existence of a left inverse for full column rank $\mathcal{H}$.

3 Blind Acquisition/Equalization

With knowledge of only the ℓ^{th} user's code $\mathbf{c}^{(\ell)}$ and the received sequence $\{r_i\}$, we consider the problem of linear estimation of the ℓ^{th} user's symbol sequence at the base station (up to arbitrary phase and symbol delay). Specifically, we generate linear symbol estimates $\hat{s}_n^{(\ell)} = \mathbf{f}^H \mathbf{r}(n)$ using receiver $\mathbf{f}$ and desire that $\hat{s}_n^{(\ell)} = \alpha_\ell s_{n-\delta_\ell}^{(\ell)}$ $\forall n$, where $|\alpha_\ell| = 1$ and $\delta_\ell \in \mathbb{Z}^+$. This is often referred to as the "blind acquisition and demodulation" problem [1] because the channels $\{\mathbf{h}^{(k)}\}$, user delays $\{\tau_k\}$, and interfering users' codes $\{\mathbf{c}^{(k)} : k \neq \ell\}$ are unknown[3] and no training sequences are transmitted.

3.1 Minimum-Entropy Acquisition

The proposed scheme can be motivated, at least on a conceptual level, by (i) the non-Gaussian nature of typical DS-CDMA sources (e.g., BPSK), and (ii) the approximate orthogonality properties of good codes, e.g. Gold codes. By "good" codes we mean those whose cross-correlation coefficients $r_{\ell,k}(i) := \sum_m c_m^{(\ell)} c_{m-i}^{(k)*}$ corresponding to user/timing mismatches are significantly smaller than those corresponding to the correct user/timing combination, i.e., $r_{\ell,\ell}(0) \gg |r_{\ell,k}(i)|$ $\forall i$, ℓ, $k \neq \ell$. For simplicity, let us first consider a situation with chip-level synchronization, equal user powers, zero inter-chip interference (ICI), and no noise. Now consider the outputs of the conventional matched-filter (MF) receiver, i.e., a receiver forming the symbol estimates $\hat{s}_n^{(\ell)} = \mathbf{c}^{(\ell)H} \mathbf{r}(n)$. When the desired user is time synchronized (i.e., $\tau_\ell = 0$), the symbol estimates take the form

$$\hat{s}_n^{(\ell)} = \underbrace{r_{\ell,\ell}(0) s_n^{(\ell)}}_{\text{non-Gaussian}} + \underbrace{\sum_{k \neq \ell}^{K} r_{\ell,k}(-\tfrac{\tau_k}{T_c}) s_n^{(k)} + \sum_{k \neq \ell}^{K} r_{\ell,k}(N - \tfrac{\tau_k}{T_c}) s_{n-1}^{(k)}}_{\text{approximately Gaussian}} . \tag{7}$$

Assuming non-Gaussian source processes and good codes, the synchronized symbol estimates are a sum of two processes: (i) a non-Gaussian desired process, and (ii) an interference process, which tends towards Gaussianity as K becomes large. When, instead, the MF receiver is *not* synchronized with the desired user, the unsynchronized symbol estimates will take the form of the latter terms in (7), i.e., approximately Gaussian. As described in Donoho's seminal paper [4], the principle of "maximizing distance from Gaussianity" is a fundamental concept that lies at the base of many blind estimation techniques that make use of higher-order statistics. Henceforth we refer to such techniques as "minimum entropy" (ME) methods in the spirit of Donoho, who demonstrated formal connections to Shannon Entropy [4].

The observations above suggest a blind timing acquisition scheme whereby the symbol-rate output streams of delay-hypothesized matched filters are examined for their "degree of Gaussianity." The delay hypothesis leading to the "least Gaussian" stream could be chosen as an estimate of the desired user's delay. The measure of Gaussianity could be constructed in various ways, e.g., using sample-average based estimates of kurtosis.

3.2 Minimum-Entropy Equalization

The previous section focused on the estimation of the desired user's chip delay, i.e., the blind *acquisition* problem. Our goal, however, is really the estimation of desired user

[3] Though the base station typically demodulates *all* in-cell users, and thus knows more than one code, the assumption of unknown interfering user codes supports a model including out-of-cell users.

symbols in the presence of unknown multipath and interference. Thus, the problem of interest bears close resemblance to a blind *equalization* problem.

Connections to minimum entropy have been established for several popular blind equalization algorithms. For example, the Shalvi-Weinstein (SW) criterion [6] minimizes the kurtosis of the estimates $\hat{s}_n$, defined $K(\hat{s}_n) := \mathrm{E}\{|\hat{s}_n|^4\} - 2\,\mathrm{E}^2\{|\hat{s}_n|^2\} - |\,\mathrm{E}\{\hat{s}_n^2\}|^2$, subject to the power constraint $\mathrm{E}\{|\hat{s}_n|^2\} = \mathrm{E}\{|s_n|^2\}$. Kurtosis is, in fact, a direct measure of "distance from Gaussianity," and it may not be surprising that the SW criterion is nearly identical to Wiggin's ME method discussed in [4]. Furthermore, it has been shown that (in the absence of noise) the minima of the constant modulus (CM) criterion [7] correspond to SW minima, regardless of the rank of $\mathcal{H}$ [8]. Thus, the CM algorithm (CMA) [9] can also be considered a member of the ME family.

A well-known difficulty in applying ME-based algorithms (such as CMA) to multiuser demodulation is that the typical "cost function does not distinguish between desired and interfering symbols, which leads to a number of local minima" [1]. As a partial solution, multiuser versions of CMA have been proposed that consist of parallel single-user demodulators whose outputs are constrained to be mutually uncorrelated (e.g., [10]). Such techniques, however, do not address the problem of locking onto a *specific* user, which is the goal of this paper.

When using an iterative ME equalization technique, the problem of demodulating a specific user reduces to the problem of *properly initializing the blind algorithm.* When the timing of the desired DS-CDMA user is known, the initialization of CMA, for example, appears rather straightforward (see, e.g., [11]). When the desired user timing is unknown, we propose initialization based on blind ME acquisition, as described in Section 3.1. In fact, we consider the goal of blind acquisition not to be the estimation of the desired user's time delay (or group delay in the presence of multipath), but rather the achievement of a good initialization for subsequent blind equalization.

In a power disparate, or "near-far," scenario, the convergence of ME equalization algorithms becomes highly sensitive to choice of initialization, making acquisition of a specific user potentially difficult. For example, the sizes of CMA's regions of convergence (in equalizer parameter space) are proportional to the eigenvalues of the autocorrelation matrix $\mathbf{R}_{rr} := \mathrm{E}\{\mathbf{r}(n)\mathbf{r}^H(n)\}$ [16]. Since weak user transmissions are characterized by signal spaces with small eigenvalues, the size of weak users' convergence regions will also be small and determining a CMA initialization within of one of these small regions may be practically impossible. This phenomenon is demonstrated in Figure 3, where not even initialization at the (global) MMSE solution was adequate for successful convergence of the two weakest users.

In an attempt to mitigate these near-far effects, we propose a whitening of the received signal prior to ME processing. Such "orthogonalization" is commonly used in many blind source separation algorithms [13] as well as some blind equalization algorithms (e.g., [6, 14]). Pre-whitening has the effect of nearly equating each user's region-of-convergence volume, significantly reducing initialization sensitivity.

4 A CMA-based Acquisition/Equalization Algorithm

A CMA-based acquisition/equalization algorithm is presented below:

1) Obtain estimate $\hat{\mathbf{T}}$ of the $L_r \times L_r$ whitening matrix $\mathbf{T} = \mathbf{V}\mathbf{\Lambda}^{-1/2}\mathbf{V}^H$, where $\mathbf{V}\mathbf{\Lambda}\mathbf{V}^H$ is the eigendecomposition of the received autocorrelation matrix $\mathbf{R}_{rr}$.

2) Compute $L_y \times 1$ filterbank output vectors $\mathbf{y}^{(\ell)}(n) := \mathbf{F}^{(\ell)H}\hat{\mathbf{T}}\mathbf{r}(n)$, for $n = 1, \ldots, M$ and $L_y \geq N$, where $\mathbf{f}_\nu^{(\ell)}$, the ν^{th} column of $\mathbf{F}^{(\ell)}$, equals $\mathbf{c}^{(\ell)}$ delayed by ν chips.

3) Estimate the normalized kurtosis of each filter output: $\hat{\kappa}_\nu^{(\ell)} := \frac{M\sum_{n=1}^{M}|y_\nu^{(\ell)}(n)|^4}{(\sum_{n=1}^{M}|y_\nu^{(\ell)}(n)|^2)^2}$.

4) Set initial equalizer $\mathbf{f}^{(\ell)}(0)$ equal to the column of $\mathbf{F}^{(\ell)}$ generating the smallest $\hat{\kappa}_\nu^{(\ell)}$.

5) Estimate ℓ^{th} user's symbols using "pre-whitened CMA": set $\mathbf{x}(n) = \hat{\mathbf{T}}\mathbf{r}(n)$, $\hat{s}_n^{(\ell)} = \mathbf{f}^{(\ell)H}(n)\mathbf{x}(n)$, and update via $\mathbf{f}^{(\ell)}(n+1) = \mathbf{f}^{(\ell)}(n) - \mu\mathbf{x}(n)\hat{s}_n^{(\ell)*}(|\hat{s}_n^{(\ell)}|^2 - 1)$.

Comment 4.1 *(Delay hypotheses):* For simplicity, 2) suggests L_y chip-spaced delay hypotheses, though better performance should result from a denser delay set (as in [1]).

Comment 4.2 *(Choice of equalization algorithm):* CMA is often considered the most widely implemented and analyzed blind equalization technique and is noted for its robustness and simplicity. Hence, it was chosen for our ME demodulation scheme. Most importantly, perhaps, CMA is known to have close ties to MMSE minimization [7] and has been shown to asymptotically achieve global MMSE solutions when $\mathcal{H}$ is full column rank and noise is absent [7]. Furthermore, CMA is known to be robust to noise and to $\mathcal{H}$ that are not full column rank [7, 9]. If speed of convergence is thought to be a problem, faster converging versions are available [15] (with higher computational cost), though pre-whitening alone greatly improves the tracking properties of CMA [14].

Comment 4.3 *(Initialization criterion):* The proposed algorithm uses kurtosis as a "measure of entropy" in the initialization stage. Recall from [8] that kurtosis-based initialization and CMA adaptation form a natural pair: $\kappa_\nu^{(\ell)} < 2$ ensures that the peak index of the asymptotic system response $\mathbf{q}^{(\ell)} := (\mathbf{f}^{(\ell)H}\hat{\mathbf{T}}\mathcal{H})^t$ equals the peak index of the initial system response. This implies that kurtosis can be directly related to acquisition probability when using codes with good cross-correlation properties.

Other initialization criteria might also work well. For example, [4] notes Gray's criterion: $\frac{M\sum_{n=1}^{M}|y_\nu(n)|^2}{(\sum_{n=1}^{M}|y_\nu(n)|)^2}$, which is easier to compute than $\hat{\kappa}_\nu^{(\ell)}$, but which simulations suggest gives comparable acquisition performance (see Figure 6).

Comment 4.4 *(Effect of noise):* In the absence of noise, pre-whitening via $\mathbf{T} = \mathbf{R}_{rr}^{-1/2}$ will result in an effective channel matrix $\mathbf{T}\mathcal{H}$ that is perfectly orthogonal. In this case, CMA regions of convergence are uniform in size and shape [16]. In the presence of noise, there is still evidence to suggest that whitening adequately reduces CMA initialization sensitivity. The pre-whitened noisy CM cost function can be written $J_{\rm cm} = -2\|\mathbf{f}^H\mathbf{R}_{rr}^{-1/2}\mathcal{H}\|_4^4 + 3\|\mathbf{f}\|_2^4 - 2\|\mathbf{f}\|_2^2 + 1$ (for BPSK), where, remarkably, the effect of noise appears only in the first term. Note that the region of convergence sizes will be determined by $\|\mathbf{f}^H\mathbf{R}_{rr}^{-1/2}\mathcal{H}\|_4^4$ since the other terms are invariant to the orientation of $\mathbf{f}$. Even when the minimum singular value of $\mathcal{H}$ is $\approx \sigma_w$, the matrix $\mathbf{R}_{rr}^{-1/2}\mathcal{H}$ will have condition number ≈ 2, implying relatively uniform regions of convergence.

A more serious problem may occur, however, when the singular values of $\mathcal{H}$ are $< \sigma_w$, since the corresponding minima of $J_{\rm cm}$ may completely disappear [12]. Unfortunately, no linear pre-processing of the received signal can remedy this. In fact, this loss of weak-user minima will plague *any* demodulation scheme based on CM cost or additive modifications thereof (e.g., [10]).

Comment 4.5 *(Effect of codes/multipath):* As the codes become less orthogonal (i.e., $|r_{\ell,k}(i)|$ increases for $k \neq \ell$ or $i \neq 0$) and/or multipath becomes more severe, it can be shown that the delayed matched-filters used as initialization hypotheses (recall algorithm steps

2-4) create output streams with higher kurtosis $\kappa_\nu^{(\ell)}$. Interpreting [8] in a CDMA context, increasing $\kappa_\nu^{(\ell)}$ above 2 decreases the probability of CMA's convergence to a user with the desired code and delay ν. (Actually, this is advantageous in preventing CMA capture by an out-of-cell interferer with the desired code, since we expect out-of-cell signals to be more significantly corrupted by multipath than in-cell signals.) In addition, higher $\kappa_\nu^{(\ell)}$ increase the probability of attraction by CM saddle points, thereby potentially slowing initial convergence [9].

Comment 4.6 *(Recursive estimation of* $\mathbf{T}$*):* For successful acquisition in a time-varying near-far environment, it is important that $\hat{\mathbf{T}}$ tracks signal variations. Fortunately, there exist various computationally-efficient recursive strategies for updating $\hat{\mathbf{T}}$ based on a forgetting factor (e.g., using QR factorization of $\mathbf{R}_{rr}^{-1/2}$ and Given's rotations). Though the estimation of $\mathbf{T}$ is the most computationally demanding part of the algorithm, one should realize that this burden is essentially distributed over all in-cell users.

Since, when $\hat{\mathbf{T}}(n)$ is tracking, we do not want variations in $\hat{\mathbf{T}}(n)$ to adversely effect the adaptation of CMA, we recommend using CMA to adapt $\mathbf{v}^{(\ell)}(n) := \hat{\mathbf{T}}^H(n)\mathbf{f}^{(\ell)}(n)$ while operating directly on the received data: $\hat{s}_n^{(\ell)} = \mathbf{v}^{(\ell)H}(n)\mathbf{r}(n)$. The resulting filter update, $\mathbf{v}^{(\ell)}(n+1) = \mathbf{v}^{(\ell)}(n) - \mu\widehat{\mathbf{R}_{rr}^{-1}}(n)\mathbf{r}(n)\hat{s}_n^{(\ell)*}(|\hat{s}_n^{(\ell)}|^2-1)$, is Gooch's "orthogonal-CMA" [14], first proposed for synchronous CDMA applications in [11].

5 Simulation Results

In all simulations, we use Gold codes with $N=31$, equalizers of length 62, user delays τ_k uniformly distributed over $[0,T)$, root-raised-cosine pulse shaping with excess bandwidth 0.2, and discrete ray multipath channels $g^{(k)}(t)$. The N_g multipath rays are normally distributed in amplitude (relative to a zero-delay ray) with std. dev. σ_g and are uniformly distributed in time over the interval $[0,T_g)$. In Figures 3-4, we assume 25 in-cell users and zero out-of-cell users, while in Figures 5-6 we assume 20 in-cell users and 120 out-of-cell users, the latter with 10 dB lower average power. For in-cell users we use $N_g=3$, $T_g=2$, and $\sigma_g=0.3$, while for out-of-cell users $N_g=10$, $T_g=10$, and $\sigma_g=0.5$. With these parameters, the in-cell and out-of-cell ICI span approximately 10 and 18 T_c, respectively. Unless otherwise noted, we assume an in-cell log-normal user power distribution with std. dev. $\sigma_p=5$ dB and out-of-cell with $\sigma_p=1$ dB. The noise $\{w_i\}$ is zero-mean AWGN with SNR = 20 dB (referenced to the average in-cell user's power).

Figures 4-6 are the result of 500 Monte Carlo simulations. In Figure 4, we compare the performance of the proposed ME scheme with Madhow's multipath-specific CMOE scheme [1, Remark 3.10][4], where we assume perfect knowledge of the received signal statistics $\mathbf{R}_{rr}$ and $\kappa_\nu^{(\ell)}$. Our performance measures include both lock probability and mean-squared symbol estimation error. "Lock" occurs when final multiuser system responses $\mathbf{q}^{(\ell)}$ achieve their absolute maximum at an index corresponding to the desired user, as discussed in Comment 4.2. MSE calculations[5] are based only on receivers achieving lock. Figure 5, which includes significant out-of-cell interference, compares the performance of the ME and CMOE schemes over a range of in-cell user power devia-

[4] We were unable to compare to [2] and [3] since both schemes require the existence of a decorrelating receiver. As discussed in Section 2.2, this is not possible in the single-sensor multi-cell environment since the anticipated number of in- plus out-of-cell users will prevent $\mathcal{H}$ from reaching full column rank.

[5] The MSE performance of the (adaptive) ME algorithm could be improved by reducing the step-size μ, whereas the performances reported for CMOE were computed in closed form.

tions. Figure 6 investigates lock probability as a function of the data length M used in estimation of initialization hypotheses (recall Comment 4.3).

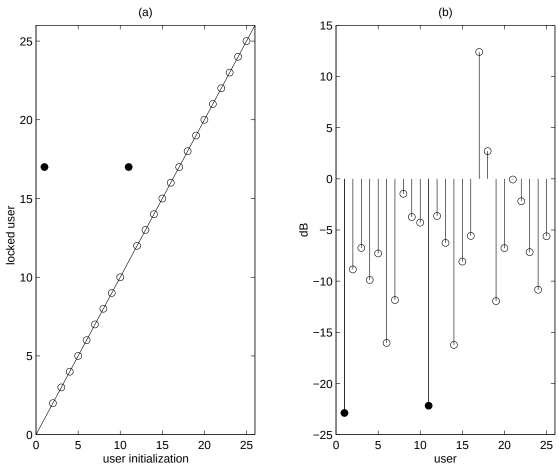

Figure 3: CMA weak-user misconvergence: (a) CMA lock from Wiener initialization, (b) user powers. (Lost users shown bold.)

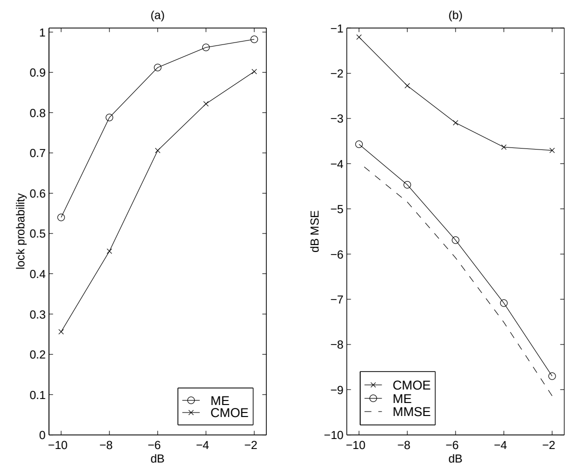

Figure 4: Lock probability and MSE performance versus desired user power (relative to 24 other equal-power users).

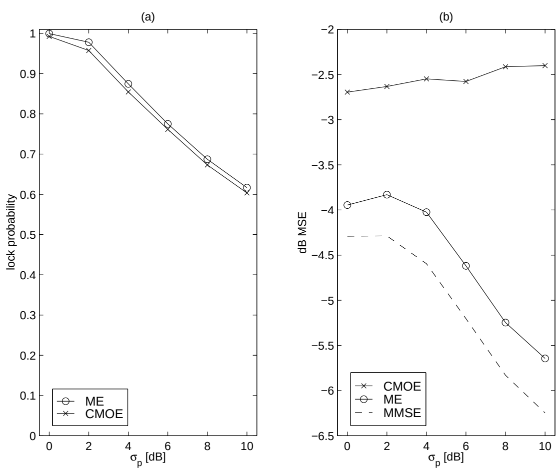

Figure 5: Lock probability and MSE performance versus in-cell (log-normal) user power deviation σ_p.

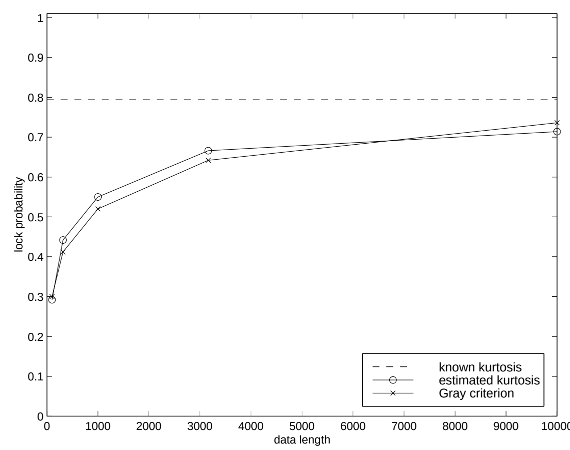

Figure 6: ME Lock probability versus data length for initialization based on data-estimated kurtosis and Gray's criterion.

6 Conclusion

We have presented a general approach to the blind acquisition/equalization of short-code CDMA based on a minimum entropy principle and, within this context, have proposed a scheme using kurtosis-based initialization of pre-whitened CMA. Simulations including multipath and significant out-of-cell interference suggest that the CMA-based method achieves higher acquisition probability and performance significantly closer to global MMSE solutions than do previously proposed schemes based on subspace-constrained optimization. Furthermore, the CMA-based approach does not require the use of multiple sensors, as do previously proposed schemes employing subspace decomposition.

Interesting questions remain concerning the relative optimality of various ME schemes and their performance in non-stationary environments.

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