Adaptive Neural/Fuzzy Control for Interpolated Nonlinear Systems

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***Abstract*—Adaptive control for nonlinear time-varying systems is of both theoretical and practical importance. In this paper, we propose an adaptive control methodology for a class of nonlinear systems with a time-varying structure. This class of systems is composed of interpolations of nonlinear subsystems which are input–output feedback linearizable. Both indirect and direct adaptive control methods are developed, where the spatially localized models (in the form of Takagi–Sugeno fuzzy systems or radial basis function neural networks) are used as online approximators to learn the unknown dynamics of the system. Without assumptions on rate of change of system dynamics, the proposed adaptive control methods guarantee that all internal signals of the system are bounded and the tracking error is asymptotically stable. The performance of the adaptive controller is demonstrated using a jet engine control problem.**

***Index Terms*—Adaptive control, nonlinear systems, online approximation, stability analysis.**

I. INTRODUCTION

ADAPTIVE control has been employed in situations where little *a priori* knowledge of the plant is known. Adaptive control has also been used to compensate for online system parameter variations, which may arise due to changes in operating points, component faults, plant deterioration, etc. The general methodology of adaptive control for time-varying systems is to treat the effects of parameter variations as unmodeled perturbations so that it turns into a robustness problem [1]. This methodology has been applied to linear time-varying systems, where the parameters vary slowly and smoothly, or discontinuously (i.e., jumps) but the discontinuities occur over large intervals of time [2]–[4]. Adaptive control for nonlinear time-varying systems has also been studied by some researchers. In [5], the authors studied adaptive control for a class of nonlinear time-varying systems in the strict feedback form with unknown unmodeled time-varying parameters or disturbances (whose bounds are known) and used the backstepping design method. Similar work has also been presented in [6] and [7]. Besides controlling the nonlinear time-varying system as a whole, another control methodology is to exploit the internal time-varying structure of nonlinear systems, for instance, the class of systems consisting of an interpolation of nonlinear dynamic equations in the strict feedback form and construct backstepping control laws tailored to each of the dynamic components of the nonlinear system [8], [9].

Note that for the adaptive control problem of nonlinear time-varying systems, only a class of systems in the strict feedback form are considered and only limited results exist so far. In this paper, we consider a more general class of nonlinear time-varying systems, which is input-output feedback linearizable and present stable adaptive control approaches using the online learning capabilities of radial basis function neural networks. This class of systems is large enough so that it is not only of theoretical interest but also of practical applicability. The idea of using function approximation structures with universal approximation properties (such as neural networks or fuzzy systems) to deal with arbitrary continuous nonlinearities has been widely used in adaptive control for nonlinear systems [10]. In fact, online approximation-based stable-adaptive neural/fuzzy methods have been significantly impacted by the work in [11]–[15] using neural networks as approximators of nonlinear functions, the work in [16]–[20] using fuzzy systems for the same purpose, and the work in [11], and [12] using dynamical neural networks. The neural and fuzzy approaches are most of the time equivalent, differing between each other mainly in the structure of the approximator chosen. Indeed, to try to bridge the gap between the neural and fuzzy approaches several researchers (e.g., in [20]) introduce adaptive schemes using a class of parameterized functions that include both neural networks and fuzzy systems. As to the approximator structure, linear in the parameter approximators are used in [13], [16], [19], [20], and nonlinear in [12], [14], [15]. Finally, most of the papers [11]–[19] deal with indirect adaptive control (trying first to identify the dynamics of the systems and then generating a control input according to the certainty equivalence principle), whereas very few authors (e.g., [20] and [21]) face the direct approach (directly generating the control input to guarantee stability), because it is not always clear how to construct the control law without knowledge of the system dynamics.

In this paper, we present an adaptive control methodology for a class of nonlinear systems that depends on exogenous scheduling variables. This class of systems consists of interpolations of nonlinear dynamic equations in feedback linearizable form and it may represent systems with a time-varying nonlinear structure, which is, indeed, a generalization of the class of feedback linearizable systems traditionally considered in nonlinear adaptive control literature [20], [22], [23]. To generalize stable adaptive fuzzy/neural control [20], following the general approach in [8] and [9], the adaptive laws applied here are

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localized in the sense that only the part of the approximator parameters corresponding to the region of the "scheduling space" is updated every time. Furthermore, besides indirect adaptive control, we also design and analyze direct adaptive control in this paper, which usually shows better transient behavior because it seems to learn and adapt faster (probably due to the fact that it has less parameters to be tuned). Both indirect and direct adaptive control methods developed here are, to our knowledge, the first of their kinds in this context.

To approach the nonlinear control problem by studying simplified, localized approximations of the plant, the control methodology studied here shares some common views of gain scheduling control, which deals with nonlinear systems that are linearized along reference trajectories or operating points [24], [25]. Gain scheduling control is widely used in industrial applications, but so far only local stability results exist due to the difficulty of stability analysis. Other related results exist in the parallel distributed compensation literature [26], [27], where authors assume the existence of a model consisting of interpolations of linearized system dynamics (which are valid within a subset of the state space or other relevant operating space). Linear controllers are designed within these regions and then interpolated with the same interpolation structure of the system. The class of time-varying systems studied in this paper is related to the system studied in the parallel distributed compensation literature. However, instead of interpolating controllable linear systems (obtained by local linearization within divided subspaces), in the spirit of [8] and [9] (but not restricted to the strict feedback form), here, we focus on using feedback linearizable nonlinear systems as the "pieces" to form the "global" nonlinear system by interpolation. Note that to achieve global stability of parallel distributed compensation approach, a single positive definite matrix that simultaneously stabilizes all combinations of linear subsystems needs to be found (which is not trivial), usually, relying on linear matrix inequalities (LMIs) optimization methods. By interpolating feedback linearizable nonlinear systems, not only can LMI optimization be avoided, but also the adaptation mechanism can be incorporated so as to guarantee asymptotic stability and deal with model uncertainty. Furthermore, this class of system is large enough (compared to the nonlinear system in the strict feedback form as studied in [8] and [9]) so that it may have more practical applicability. This will be shown via our jet engine example where a model in the strict feedback form (or an interpolation of such models) cannot be used to adequately represent the engine, whereas our feedback linearizable model can do this quite well.

This paper is organized as follows. The spatially localized model architecture of radial basis function neural networks and Takagi–Sugeno fuzzy systems is discussed in Section II, which serve as *linear in the parameter* online approximators. In Section III, the details of the problem formulation for a class of input-output feedback linearizable time-varying nonlinear systems are given. The adaptive algorithms and system stability analysis are presented in Sections IV and V for both indirect and direct schemes. Section VI describes the application to a jet engine control problem to illustrate the performance of the proposed adaptive neural/fuzzy control methods.

II. Spatially Localized Model Architecture

In neurobiological studies, the concept of localized information processing in the form of receptive fields has been known and demonstrated by experimental evidence (e.g., locally tuned and overlapping receptive fields have been found in parts of the cerebral cortex, in the visual cortex and in other parts of the brain), which suggests that such local learning offers alternative computational opportunities to learning with "global basis functions," such as the multilayer perceptron neural network with sigmoidal activation functions [28]. Inspired by these biological counterparts, the radial basis function neural network model has been presented, which can be defined by

$$y = F_{rbf}(x, \theta) = \sum_{i=1}^{M} b_i R_i(x) \tag{1}$$

where y is the output of the radial basis function network, $x = [x_1, x_2, \ldots, x_n]^\top$ holds the n inputs and $i = 1, 2, \ldots, M$ represent M receptive field units. The vector θ holds the parameters of the "receptive field units," which consist of the "strength" parameters b_i and possibly the parameters of the "radial basis functions" $R_i(x)$ (a.k.a., radial response functions or kernel functions, defining the activation extents of the corresponding receptive fields with the characteristics that their responses decrease monotonically with distance from a central point). There are several possible choices for the receptive field functions $R_i(x)$. Typically, Gaussian-shaped functions are used for analytical convenience, that is

$$R_i(x) = \exp\left(-\frac{1}{2}(x - c_i)^\top D_i (x - c_i)\right) \tag{2}$$

where $c_i = [c_1^i, c_2^i, \ldots, c_n^i]^\top$ parameterize the locations of the receptive fields in the input space and $D_i = diag((1/\sigma_1^i)^2, (1/\sigma_2^i)^2, \ldots, (1/\sigma_n^i)^2)$ determine the shapes (or relative widths) of the receptive fields. Note that rather than computing the output of the radial basis function network with the simple sum as in (1), there are also alternatives, for instance, by computing a weighted average

$$y = F_{rbf}(x, \theta) = \frac{\sum_{i=1}^{M} b_i R_i(x)}{\sum_{i=1}^{M} R_i(x)}. \tag{3}$$

Moreover, to improve modeling flexibility of the radial basis function networks, it is also possible to further define the strength parameters b_i to be parametric functions

$$b_i(x) = a_{i,0} + a_{i,1}x_1 + \cdots + a_{i,n}x_n \tag{4}$$

where $a_{i,j}$, $i = 1, 2, \ldots, M$, and $j = 1, 2, \ldots, n$ are strength function parameters.

Another type of spatially localized model is the Takagi–Sugeno fuzzy system. The fuzzy system facilitates the emulation of human intelligence by modeling human cognitive processes in the form of rules and inference mechanisms. Abstracted from the qualitative description of premise representation, inference and defuzzification, the mathematical formula of the Takagi–Sugeno fuzzy system can be defined by

$$y = F_{ts}(x,\theta) = \frac{\sum_{i=1}^{R} g_i(x)\mu_i(x)}{\sum_{i=1}^{R} \mu_i(x)} \tag{5}$$

$$g_i(x) = a_{i,0} + a_{i,1}x_1 + \cdots + a_{i,n}x_n \tag{6}$$

$$\mu_i(x) = \prod_{j=1}^{n} \exp\left(-\frac{1}{2}\left(\frac{x_j - c_j^i}{\sigma_j^i}\right)^2\right) \tag{7}$$

where y is the output of the fuzzy system, $x = [x_1, x_2, \ldots, x_n]^\top$ holds the n inputs, and $i = 1, 2, \ldots, R$ represent R different rules [29]. The shapes of the membership functions are chosen to be Gaussian and center-average defuzzification and product are used for the premise and implication in the structure of the fuzzy system. The $g_i(x)$, $i = 1, 2, \ldots, R$ are called consequent functions of the fuzzy system, where the $a_{i,j}$ are the parameters. The premise membership functions $\mu_i(x)$ are assumed to be well-defined so that $\Sigma_{i=1}^{R}\mu_i(x) \neq 0$ for all x. The parameters c_j^i and σ_j^i are the centers and relative widths of the membership functions, respectively, for the jth inputs and the ith rules.

Actually, the radial basis function neural networks previously described (2), (3), and (4) are functionally equivalent to the Takagi–Sugeno fuzzy systems defined by (5), (6), and (7). To see this, suppose that we let the number of receptive field units equal to the number of rules (i.e., $M = R$), let the receptive field strength functions same as the consequent functions (i.e., $b_i(x) = g_i(x)$) and choose the parameters of the radial basis functions same as those of the premise membership functions (i.e., $R_i(x) = \mu_i(x)$). In this case, the radial basis function network is identical to the Takagi–Sugeno fuzzy system. Note that the tunable parameter vector θ in (3) or (5) can be composed of both radial basis function (or premise membership function) parameters c_j^i and σ_j^i and strength function (or consequent function) parameters $a_{i,j}$. This is referred to as *nonlinear in the parameter* approximator. A *nonlinear in the parameter* spatially localized model can be tuned by a variety of gradient methods such as the steepest descent method and Levenberg–Marquardt method. Alternatively, we may decompose the parameter vector into a linear part θ_1 consisting of the strength (or consequent) function parameters and a nonlinear part θ_2 composed of the radial basis function (or premise membership function) parameters. By having the tunable parameter vector θ be composed of $a_{i,j}$ only and specifying the parameters c_j^i and σ_j^i in advance, we will have a *linear in the parameter* radial basis function network (or Takagi–Sugeno fuzzy system)

$$y = F_{rbf}(x,\theta) = F_{ts}(x,\theta) = \theta^\top \phi(x). \tag{8}$$

Note that the *linear in the parameter* radial basis function networks or Takagi–Sugeno fuzzy systems also have the capabilities of forming an arbitrarily accurate approximation to any continuous nonlinear function, so that in the following adaptive control mechanisms, we will use them as online approximators to learn the unknown dynamics of the system. This will facilitate the derivation of adaptive laws and the analysis of system stability.

III. A Class of Nonlinear Systems With a Time-Varying Structure

Consider a class of nonlinear systems consisting of an interpolation of nonlinear subsystems

$$\dot{x} = \left(\sum_{i=1}^{N} f^i(x,v)\xi^i(v)\right) + \left(\sum_{i=1}^{N} g^i(x,v)\xi^i(v)\right) u \tag{9}$$

$$y = \sum_{i=1}^{N} h^i(x,v)\xi^i(v) \tag{10}$$

where $x = [x_1, x_2, \ldots, x_n]^\top$ is the (measurable) state vector, u is the (scalar) input, and y is the (scalar) output of the system. The functions $f^i(x,v)$, $g^i(x,v)$ and $h^i(x,v)$ represent smooth local nonlinear dynamics. The functions $\xi^i(v)$ ($\xi^i(v) > 0$) are smooth interpolation functions, $i = 1, 2, \ldots, N$ split the domain of v into N different nonlinear subregions and v is a vector of exogenous scheduling variables which are measurable and bounded.

Let $L_g^d h^i(x,v)$ be the dth Lie derivative of $h^i(x,v)$ with respect to $g(x,v) = \sum_{j=1}^{N} g^j(x,v)\xi^j(v)$, that is, for example, $L_g h^i(x,v) = ((\partial h^i(x,v))/\partial x)^\top \sum_{j=1}^{N} g^j(x,v)\xi^j(v)$, $L_g^2 h^i(x,v) = L_g[L_g h^i(x,v)]$, and so on. For convenience of derivation, we define $\bar{L}_f^d h^i(x,v)$ to be the dth *modified* Lie derivative of $h^i(x,v)$ with respect to $f(x,v) = \sum_{j=1}^{N} f^j(x,v)\xi^j(v)$, that is

$$\bar{L}_f h^i(x,v) = \frac{h^i(x,v)\dot{\xi}^i(v)}{\xi^i(v)} + \left(\frac{\partial h^i(x,v)}{\partial v}\right)^\top \dot{v} + \left(\frac{\partial h^i(x,v)}{\partial x}\right)^\top \sum_{j=1}^{N} f^j(x,v)\xi^j(v)$$

and, for example, $\bar{L}_f^2 h^i(x,v) = \bar{L}_f[\bar{L}_f h^i(x,v)]$. Next, we give the definition of the "strong relative degree," that is, a system is said to have a strong relative degree d ($1 \leq d \leq n$) if

$$\begin{aligned}\sum_{i=1}^{N} L_g h^i(x,v)\xi^i(v) &= \sum_{i=1}^{N} L_g \bar{L}_f h^i(x,v)\xi^i(v)\\ &= \cdots = \sum_{i=1}^{N} L_g \bar{L}_f^{d-2} h^i(x,v)\xi^i(v)\\ &= 0\end{aligned}$$

and $\sum_{i=1}^{N} L_g \bar{L}_f^{d-1} h^i(x,v)\xi^i(v) \neq 0$ for all x and v. Note that we use both the standard and *modified* Lie derivatives to provide a compact representation here.

Under the aforementioned definitions, if the system represented by (9) and (10) has a strong relative degree d, then

$$\dot{y} = \sum_{i=1}^{N} \left[h^i(x,v)\dot{\xi}^i(v) + \left(\frac{\partial h^i(x,v)}{\partial x}\right)^\top \dot{x}\xi^i(v)\right.$$

$$
\begin{aligned}
&\quad + \left(\frac{\partial h^i(x,v)}{\partial v}\right)^\top \dot{v}\xi^i(v)\Bigg] \\
&= \sum_{i=1}^N \Bigg[h^i(x,v)\dot{\xi}^i(v) + \left(\frac{\partial h^i(x,v)}{\partial v}\right)^\top \dot{v}\xi^i(v) \\
&\quad + \left(\frac{\partial h^i(x,v)}{\partial x}\right)^\top \sum_{j=1}^N f^j(x,v)\xi^j(v)\xi^i(v)\Bigg] \\
&\quad + \sum_{i=1}^N \left[\left(\frac{\partial h^i(x,v)}{\partial x}\right)^\top \sum_{j=1}^N g^j(x,v)\xi^j(v)\xi^i(v)\right] u \\
&= \sum_{i=1}^N \bar{L}_f h^i(x,v)\xi^i(v)
\end{aligned}
$$

and so on, so that the system dynamics may be written in the normal form as

$$
\begin{aligned}
\dot{\zeta}_1 &= \zeta_2 = \sum_{i=1}^N \bar{L}_f h^i(x,v)\xi^i(v) \\
\dot{\zeta}_2 &= \zeta_3 = \sum_{i=1}^N \bar{L}_f^2 h^i(x,v)\xi^i(v) \\
&\vdots \\
\dot{\zeta}_{d-1} &= \zeta_d = \sum_{i=1}^N \bar{L}_f^{d-1} h^i(x,v)\xi^i(v) \\
\dot{\zeta}_d &= \sum_{i=1}^N \bar{L}_f^d h^i(x,v)\xi^i(v) + \sum_{i=1}^N L_g \bar{L}_f^{d-1} h^i(x,v)\xi^i(v)u \\
\dot{\pi} &= f_0(\zeta,\pi,v)
\end{aligned}
$$

with $\zeta \in \Re^d$, $\pi \in \Re^{n-d}$ and $\zeta_1 = y$. Note that the class of nonlinear systems we consider here is actually a special case of a general class of nonlinear time-varying systems

$$
\begin{aligned}
\dot{x} &= f(x,t) + g(x,t)u && (11) \\
y &= h(x,t) && (12)
\end{aligned}
$$

and there exists certain kind of equivalence between the Lie derivatives previously defined and the ones for the time-varying systems [30], [31]. In particular, we have

$$
\begin{aligned}
L_g h(x,t) &= \left(\frac{\partial h}{\partial x}\right)^\top g(x,t) \\
&= \sum_{i=1}^N \left(\frac{\partial h^i(x,v)}{\partial x}\right)^\top g(x,t)\xi^i(v) \\
&= \sum_{i=1}^N L_g h^i(x,v)\xi^i(v)
\end{aligned}
$$

and

$$
\begin{aligned}
\bar{L}_f h(x,t) &= \frac{\partial h}{\partial t} + \left(\frac{\partial h}{\partial x}\right)^\top f(x,t) \\
&= \left(\frac{\partial h}{\partial v}\right)^\top \dot{v} + \left(\frac{\partial h}{\partial x}\right)^\top f(x,t) \\
&= \sum_{i=1}^N \left(\frac{\partial \left[h^i(x,v)\xi^i(v)\right]}{\partial v}\right)^\top \dot{v} \\
&\quad + \sum_{i=1}^N \left(\frac{\partial h^i(x,v)}{\partial x}\right)^\top \sum_{j=1}^N f^j(x,v)\xi^j(v)\xi^i(v) \\
&= \sum_{i=1}^N \frac{h^i(x,v)\dot{\xi}^i(v)}{\xi^i(v)}\xi^i(v) + \sum_{i=1}^N \left(\frac{\partial h^i(x,v)}{\partial v}\right)^\top \dot{v}\xi^i(v) \\
&\quad + \sum_{i=1}^N \left(\frac{\partial h^i(x,v)}{\partial x}\right)^\top \sum_{j=1}^N f^j(x,v)\xi^j(v)\xi^i(v) \\
&= \sum_{i=1}^N \bar{L}_f h^i(x,v)\xi^i(v)
\end{aligned}
$$

so that the previous normal form can be rewritten as

$$
\begin{aligned}
\dot{\zeta}_1 &= \zeta_2 = \bar{L}_f h(x,t) \\
\dot{\zeta}_2 &= \zeta_3 = \bar{L}_f^2 h(x,t) \\
&\vdots \\
\dot{\zeta}_{d-1} &= \zeta_d = \bar{L}_f^{d-1} h(x,t) \\
\dot{\zeta}_d &= \bar{L}_f^d h(x,t) + L_g \bar{L}_f^{d-1} h(x,t)u \\
\dot{\pi} &= f_0(\zeta,\pi,t)
\end{aligned}
$$

which is the same as the normal form of the time-varying systems [30], [31]. Therefore, there exists the diffeomorphism so that the normal form can be obtained from (9) and (10) by a change of variables.

Although the class of nonlinear systems studied here is a special case of the general time-varying systems, the advantage of using this interpolated form is to exploit the internal structure of the time-varying dynamics so that they can be separated into known scheduling functions $\xi^i(v)$ (which could be fast time-varying) and unknown local nonlinear dynamics consisting of $f^i(x,v)$, $g^i(x,v)$, and $h^i(x,v)$. By using an interpolated online approximation-based adaptive control strategy, we expect that the local nonlinear dynamics may be approximated more accurately by corresponding online local approximators so that the performance of adaptive control can be improved, of course, at the cost of the increase of computational complexity. Moreover, by using the scheduling functions to explicitly represent the known but fast time-varying dynamics as the interpolation between local subsystems, the interpolated adaptive controller is expected to handle a class of fast time-varying systems without assumption on rate of change of system dynamics.

The normal form decomposes the system states into an external part ζ and an internal part π. For the external part, if we let $y^{(d)}$ denote the dth derivative of y, it can be rewritten as

$$
\begin{aligned}
y^{(d)} &= \sum_{i=1}^N \left(\alpha_k^i(t) + \alpha^i(x,v)\right)\xi^i(v) \\
&\quad + \sum_{i=1}^N \left(\beta_k^i(t) + \beta^i(x,v)\right)\xi^i(v)u \qquad (13)
\end{aligned}
$$

where $\alpha_k^i(t)$ and $\beta_k^i(t)$ are "known" local dynamics of the system (which are assumed to be bounded if x is bounded) and

$\alpha^i(x,v)$ and $\beta^i(x,v)$ represent nonlinear local dynamics of the plant that are unknown. We assume that for some known $\beta_0^i > 0$, we have $|\beta_k^i(t) + \beta^i(x,v)| \geq \beta_0^i$ so that it is always bounded away from zero (for convenience we further assume that $\beta_k^i(t) + \beta^i(x,v) > 0$, however, the following analysis may easily be modified for systems which are defined with $\beta_k^i(t) + \beta^i(x,v) < 0$ as well). The external part may be stabilized by the control u (which we will show later), while the internal part is made uncontrollable by the same control. By having $\zeta = 0$ in the inner part, the "zero dynamics" of the system are given by

$$\dot{\pi} = f_0(0,\pi,t) \tag{14}$$

If the plant is of relative degree $d = n$, then there are no zero dynamics (i.e., no internal part π). Alternatively, if the relative degree $d < n$, we assume that the zero dynamics are exponentially attractive so that if we have some control u to let ζ bounded, this also ensures boundedness of π.

IV. Indirect Adaptive Control

The online learning abilities of fuzzy systems or neural networks are considered here to approximate the unknown local dynamics of the nonlinear system. In particular, the *linear in the parameter* Takagi–Sugeno fuzzy systems (or radial basis function networks) are taking the form of

$$\hat{\alpha}^i(x,v) = \theta_\alpha^{i\top}(t)\phi_\alpha^i(x,v) \tag{15}$$
$$\hat{\beta}^i(x,v) = \theta_\beta^{i\top}(t)\phi_\beta^i(x,v) \tag{16}$$

where the parameter vectors $\theta_\alpha^i(t)$ and $\theta_\beta^i(t)$, $i = 1,2,\ldots,N$ are assumed to be defined within the compact parameter sets Ω_α and Ω_β, respectively. (Refer to Section II for an explanation on how neural networks or fuzzy systems can be put into this form.) In addition, we define the subspace $S_x \subseteq \Re^n$ as the space through which the state trajectory may travel under closed-loop control (we are making no *a priori* assumptions here about the size of S_x). Note that besides the tunable parameters contained in the vectors $\theta_\alpha^i(t)$ and $\theta_\beta^i(t)$ that are adjusted online by the update laws, it is also very important to properly specify the structure parameters such as the centers and shapes of the membership functions (or receptive fields). Although these structure parameters are defined in advance and will not affect the stability of the adaptive controller, the choice of these parameters should have a reasonable cover (e.g., with uniformly distributed centers) of the state space S_x so as to accurately approximate the system dynamics.

We also define the actual system as

$$\alpha^i(x,v) = \theta_\alpha^{i*\top}\phi_\alpha^i(x,v) + \omega_\alpha^i(x,v) \tag{17}$$
$$\beta^i(x,v) = \theta_\beta^{i*\top}\phi_\beta^i(x,v) + \omega_\beta^i(x,v) \tag{18}$$

where

$$\theta_\alpha^{i*} = \arg\min_{\theta_\alpha^i\in\Omega_\alpha}\left(\sup_{x\in S_x}\left|\theta_\alpha^{i\top}\phi_\alpha^i(x,v) - \alpha^i(x,v)\right|\right) \tag{19}$$
$$\theta_\beta^{i*} = \arg\min_{\theta_\beta^i\in\Omega_\beta}\left(\sup_{x\in S_x}\left|\theta_\beta^{i\top}\phi_\beta^i(x,v) - \beta^i(x,v)\right|\right) \tag{20}$$

are the optimal parameters and $\omega_\alpha^i(x,v)$ and $\omega_\beta^i(x,v)$ are approximation errors which arise when $\alpha^i(x,v)$ and $\beta^i(x,v)$ are represented by finite size approximators (with specific approximator structures). We assume that

$$\left|\omega_\alpha^i(x,v)\right| \leq W_\alpha^i(x) \tag{21}$$
$$\left|\omega_\beta^i(x,v)\right| \leq W_\beta^i(x) \tag{22}$$

where $W_\alpha^i(x)$ and $W_\beta^i(x)$ are known state dependent bounds on the errors in representing the actual system with approximators. This assumption is generally hold according to the universal approximation property of neural networks and fuzzy systems by properly defining the approximator structures and parameters. We also define parameter errors to be

$$\tilde{\theta}_\alpha^i(t) = \theta_\alpha^i(t) - \theta_\alpha^{i*} \tag{23}$$
$$\tilde{\theta}_\beta^i(t) = \theta_\beta^i(t) - \theta_\beta^{i*}. \tag{24}$$

We want to design a control system which will cause the output $y(t)$ and its derivatives $\dot{y}(t),\ldots,y^{(d)}(t)$ to track a desired reference trajectory $y_m(t)$ and its derivatives $\dot{y}_m(t),\ldots,y_m^{(d)}(t)$, respectively, which we assume to be bounded. The reference trajectory may be defined by a reference signal whose first d derivatives are measurable, or by any reference input $r(t)$ passing through a reference model, with relative degree equal to or greater than d. In particular, a linear reference model may be

$$\frac{Y_m(s)}{R(s)} = \frac{q(s)}{p(s)} = \frac{q_0}{s^d + p_{d-1}s^{d-1} + \cdots + p_0} \tag{25}$$

where $p(s)$ is the pole polynomial with stable roots.

The indirect adaptive control law is designed as

$$u = u_{ce} + u_{si} \tag{26}$$

which is comprised of a "certainty equivalence" control term u_{ce} and a "sliding mode" control term u_{si}

$$u_{ce} = \frac{1}{\sum_{i=1}^{N}\left[\beta_k^i(t) + \hat{\beta}^i(x,v)\right]\xi^i(v)} \cdot \left(-\sum_{i=1}^{N}\left[\alpha_k^i(t) + \hat{\alpha}^i(x,v)\right]\xi^i(v) + \nu(t)\right) \tag{27}$$
$$u_{si} = \frac{\sum_{i=1}^{N}\left[W_\alpha^i(x) + W_\beta^i(x)\left|u_{ce}\right|\right]\xi^i(v)}{\sum_{i=1}^{N}\beta_0^i\xi^i(v)} sgn(e_s). \tag{28}$$

The certainty equivalence term is used to exploit the approximated system dynamics $\hat{\alpha}^i(x,v)$ and $\hat{\beta}^i(x,v)$ to construct the feedback controller. It may also take advantage of *a priori* knowledge of system dynamics $\alpha_k^i(t)$ and $\beta_k^i(t)$ so as to simplify the unknown dynamics and facilitate the online learning process. Noting the existence of approximation inaccuracy, the sliding mode control term is introduced to compensate for approximation errors, improve system robustness and guarantee system stability.

Let the tracking error be $e(t) = y_m(t) - y(t)$ and a measure of the tracking error be $e_s(t) = e^{(d-1)}(t) + k_{d-2}e^{(d-2)}(t) + \cdots + k_1\dot{e}(t) + k_0 e(t)$, that is, in the frequency domain, $e_s(s) = L(s)e(s)$ with $L(s) = s^{(d-1)} + k_{d-2}s^{(d-2)} + \cdots + k_1 s + k_0$ whose roots are chosen to be in the (open) left-half plane. Also, for convenience, we later let $\bar{e}_s(t) = \dot{e}_s(t) - e^{(d)}(t)$. Notice that our control goal is to drive $e_s(t) \to 0$ as $t \to \infty$ and the shape of the error dynamics is dictated by the choice of the design parameters in $L(s)$. We define

$$\nu(t) = y_m^{(d)} + \eta e_s + \bar{e}_s \tag{29}$$

with $\eta > 0$ as a design parameter.

Consider the update laws

$$\dot{\theta}_\alpha^i(t) = -\,{Q_\alpha^i}^{-1}\phi_\alpha^i(x,v)e_s\xi^i(v) \tag{30}$$

$$\dot{\theta}_\beta^i(t) = -\,{Q_\beta^i}^{-1}\phi_\beta^i(x,v)e_s u_{ce}\xi^i(v) \tag{31}$$

where Q_α^i and Q_β^i are positive definite and diagonal and serve as adaptation gains for the parameter updates, and $\xi^i(v)$ represent the effects of interpolated adaptive laws, that is, the degree of parameter adaptation of the local approximator is up to the extent of involvement of that subregion indicated by $\xi^i(v)$. Note that the aforementioned adaptation laws do not guarantee that $\theta_\alpha^i \in \Omega_\alpha$ and $\theta_\beta^i \in \Omega_\beta$ so that we will use a projection method to ensure this, in particular, to make sure that $\beta_k^i(t) + \hat{\beta}^i(x,v) \geq \beta_0^i$.

It is also worthy to mention that although in the proposed interpolate d adaptive controller the number of receptive fields (or fuzzy rules) is increased as a result of dividing the system into several local subsystems, it is not the same as having one pair of global approximator (i.e., $\hat{\alpha}^1$ and $\hat{\beta}^1$) with the increased size of the parameter vectors. This is because for the proposed interpolated adaptive controller, as the sources of the fast time-varying dynamics is known and measurable, they are explicitly expressed as the scheduling variables. Thus, only local system dynamics need to be approximated (which are usually unknown but not fast time-varying) and the adaptive controller is expected to handle the fast time-varying dynamics. However, for the scheme where only one single pair of approximators is used, the information on the sources of the fast time-varying dynamics is not extracted so that the system dynamics appear to be unknown and fast time-varying in a whole, which increases the difficulty of online learning and adaptation.

Theorem 1: Consider the nonlinear system (9) and (10) with strong relative degree d. Assume that: 1) $\alpha_k^i(t)$ and $\beta_k^i(t)$ in (13) are bounded if x is bounded, 2) $\beta_k^i(t) + \beta^i(x,v) \geq \beta_0^i$ for some known $\beta_0^i > 0$, 3) $|\omega_\alpha^i(x,v)| \leq W_\alpha^i(x)$ and $|\omega_\beta^i(x,v)| \leq W_\beta^i(x)$ with known $W_\alpha^i(x)$ and $W_\beta^i(x)$, 4) $y_m(t), \dot{y}_m(t), \ldots, y_m^{(d)}(t)$ are measurable and bounded, 5) $x(t), y(t), \dot{y}(t), \ldots, y^{(d-1)}(t)$ are measurable. and 6) $1 \leq d < n$ with the zero dynamics exponentially attractive or $d = n$. Under these conditions there exist indirect adaptive control laws (26), (27), and (28) and update laws (30) and (31) such that all internal signals are bounded and the tracking error e is asymptotically stable.

Proof: Consider the Lyapunov function candidate

$$V_i = \frac{1}{2}e_s^2 + \frac{1}{2}\sum_{i=1}^N \tilde{\theta}_\alpha^{i\top} Q_\alpha^i \tilde{\theta}_\alpha^i + \frac{1}{2}\sum_{i=1}^N \tilde{\theta}_\beta^{i\top} Q_\beta^i \tilde{\theta}_\beta^i \tag{32}$$

Using vector derivatives and following [20] and [32], the time derivative of (32) is

$$\dot{V}_i = e_s\dot{e}_s + \sum_{i=1}^N \tilde{\theta}_\alpha^{i\top} Q_\alpha^i \dot{\tilde{\theta}}_\alpha^i + \sum_{i=1}^N \tilde{\theta}_\beta^{i\top} Q_\beta^i \dot{\tilde{\theta}}_\beta^i. \tag{33}$$

Note that $\bar{e}_s(t) = \dot{e}_s(t) - e^{(d)}(t)$ and the dth derivative of the output error is $e^{(d)} = y_m^{(d)} - y^{(d)}$ so that

$$\dot{e}_s(t) = \bar{e}_s(t) + y_m^{(d)} - y^{(d)} \tag{34}$$

and from (13), (26), (29), and (27), the equation shown at the bottom of the next page holds true. Also, from (15)–(18), (23), and (24), we have

$$\begin{aligned}\dot{e}_s = &-\eta e_s + \sum_{i=1}^N \left[\tilde{\theta}_\alpha^{i\top}\phi_\alpha^i(x,v) - \omega_\alpha^i(x,v)\right]\xi^i(v)\\ &+ \sum_{i=1}^N \left[\tilde{\theta}_\beta^{i\top}\phi_\beta^i(x,v) - \omega_\beta^i(x,v)\right]\xi^i(v)u_{ce}\\ &- \sum_{i=1}^N \left[\beta_k^i(t) + \beta^i(x,v)\right]\xi^i(v)u_{si}.\end{aligned}$$

Substitute the aforementioned equation into (33) and assume that the ideal parameters are constant (which is achieved by having $\omega_\alpha^i(x,v)$ and $\omega_\beta^i(x,v)$ in (17) and (18) represent approximation errors from the time-varying part of system dynamics) so that $\dot{\tilde{\theta}}_\alpha^i = \dot{\theta}_\alpha^i$ and $\dot{\tilde{\theta}}_\beta^i = \dot{\theta}_\beta^i$ and substitute (30) and (31) into (33)

$$\begin{aligned}\dot{V}_i =& e_s\dot{e}_s + \sum_{i=1}^N \tilde{\theta}_\alpha^{i\top} Q_\alpha^i \left[-{Q_\alpha^i}^{-1}\phi_\alpha^i(x,v)e_s\xi^i(v)\right]\\ &+ \sum_{i=1}^N \tilde{\theta}_\beta^{i\top} Q_\beta^i \left[-{Q_\beta^i}^{-1}\phi_\beta^i(x,v)e_s u_{ce}\xi^i(v)\right]\\ =& e_s\dot{e}_s - \sum_{i=1}^N \tilde{\theta}_\alpha^{i\top}\phi_\alpha^i(x,v)e_s\xi^i(v)\\ &- \sum_{i=1}^N \tilde{\theta}_\beta^{i\top}\phi_\beta^i(x,v)e_s\xi^i(v)u_{ce}\\ =& -\eta e_s^2 - \sum_{i=1}^N \left[\omega_\alpha^i(x,v) + \omega_\beta^i(x,v)u_{ce}\right]\xi^i(v)e_s\\ &- \sum_{i=1}^N \left[\beta_k^i(t) + \beta^i(x,v)\right]\xi^i(v)u_{si}e_s.\end{aligned}$$

Notice that we did not consider a projection modification to the previous update laws. Clearly, since $\theta_\alpha^{i*} \in \Omega_\alpha$ and $\theta_\beta^{i*} \in \Omega_\beta$, when the projection is in effect it always results in smaller parameter errors that will decrease $\dot{V}_i$ so that

$$\dot{V}_i \le -\eta e_s^2 - \sum_{i=1}^{N} \left[\omega_\alpha^i(x,v) + \omega_\beta^i(x,v)u_{ce}\right] \xi^i(v)e_s - \sum_{i=1}^{N} \left[\beta_k^i(t) + \beta^i(x,v)\right] \xi^i(v)u_{si}e_s. \quad (35)$$

Note that

$$\begin{aligned} -\left[\omega_\alpha^i(x,v) + \omega_\beta^i(x,v)u_{ce}\right]e_s &\le \left[|\omega_\alpha^i(x,v)| + |\omega_\beta^i(x,v)u_{ce}|\right] |e_s| \\ &\le \left[W_\alpha^i(x) + W_\beta^i(x)|u_{ce}|\right] |e_s| \end{aligned}$$

and $\xi^i(v) > 0$, we have

$$-\sum_{i=1}^{N} \left[\omega_\alpha^i(x,v) + \omega_\beta^i(x,v)u_{ce}\right] \xi^i(v)e_s \le \sum_{i=1}^{N} \left[W_\alpha^i(x) + W_\beta^i(x)|u_{ce}|\right] \xi^i(v)|e_s|.$$

Also, note that $\beta_k^i(t) + \beta^i(x,v) \ge \beta_0^i > 0$ so that $\sum_{i=1}^{N} \left[\beta_k^i(t) + \beta^i(x,v)\right] \xi^i(v) \ge \sum_{i=1}^{N} \beta_0^i \xi^i(v) > 0$ and, considering (28), we have

$$\begin{aligned} &-\sum_{i=1}^{N} \left[\beta_k^i(t) + \beta^i(x,v)\right] \xi^i(v)u_{si}e_s \\ &= \sum_{i=1}^{N} \left[\beta_k^i(t) + \beta^i(x,v)\right] \xi^i(v) \cdot \frac{\sum_{i=1}^{N} \left[W_\alpha^i(x) + W_\beta^i(x)|u_{ce}|\right] \xi^i(v)}{\sum_{i=1}^{N} \beta_0^i \xi^i(v)} |e_s| \\ &\le -\sum_{i=1}^{N} \left[W_\alpha^i(x) + W_\beta^i(x)|u_{ce}|\right] \xi^i(v)|e_s|. \end{aligned}$$

Thus, by substituting the aforementioned equation into (35) we have $\dot{V}_i \le -\eta e_s^2$ which means V_i is a nonincreasing function of time, so that the measure of the tracking error e_s is bounded. As $e_s(s) = L(s)e(s)$ and $L(s)$ is a stable function with the degree of $d-1$, we know that the tracking error and its derivatives $e, \dot{e}, \ldots, e^{(d-1)}$ are bounded. Since the reference trajectory y_m and its derivatives $\dot{y}_m, \ldots, y_m^{(d-1)}$ are assumed to be bounded, the system output y and its derivatives $\dot{y}, \ldots, y^{(d-1)}$ are bounded. Hence, ξ is bounded and, thus, x is bounded. Besides, the fact that $\dot{V}_i$ is negative semidefinite also implies that parameter estimations θ_α^i and θ_β^i are bounded. Therefore, the boundedness of $\hat{\alpha}^i(x,v)$, $\hat{\beta}^i(x,v)$, $\alpha_k^i(t)$ and $\beta_k^i(t)$ assures that u_{ce} and u_{si} and hence u are bounded.

To show asymptotic stability of the output, note that

$$\int_0^\infty \eta e_s^2 dt \le -\int_0^\infty \dot{V}_i dt = V_i(0) - V_i(\infty) \quad (36)$$

this establishes that $e_s \in L_2$ ($L_2 = \{z(t) : \int_0^\infty z^2(t) < \infty\}$) since $V_i(0)$ and $V_i(\infty)$ are bounded. Since e_s and $\dot{e}_s$ are bounded and $e_s \in L_2$, by Barbalat's Lemma, we have asymptotic stability of e_s, which implies asymptotic stability of the tracking error e (i.e., $\lim_{t\to\infty} e = 0$). □

$$\begin{aligned} \dot{e}_s(t) =& \bar{e}_s(t) + (\nu(t) - \eta e_s - \bar{e}_s) \\ &- \left(\sum_{i=1}^{N} \left[\alpha_k^i(t) + \alpha^i(x,v)\right] \xi^i(v) + \sum_{i=1}^{N} \left[\beta_k^i(t) + \beta^i(x,v)\right] \xi^i(v)(u_{ce} + u_{si})\right) \\ =& -\eta e_s + \left(\nu(t) - \sum_{i=1}^{N} \alpha_k^i(t)\xi^i(v) - \sum_{i=1}^{N} \left[\beta_k^i(t) + \hat{\beta}^i(x,v)\right] \xi^i(v)u_{ce}\right) - \sum_{i=1}^{N} \alpha^i(x,v)\xi^i(v) \\ &+ \sum_{i=1}^{N} \left[\hat{\beta}^i(x,v) - \beta^i(x,v)\right] \xi^i(v)u_{ce} - \sum_{i=1}^{N} \left[\beta_k^i(t) + \beta^i(x,v)\right] \xi^i(v)u_{si} \\ =& -\eta e_s + \left(\nu(t) - \sum_{i=1}^{N} \alpha_k^i(t)\xi^i(v) + \sum_{i=1}^{N} \left[\alpha_k^i(t) + \hat{\alpha}^i(x,v)\right] \xi^i(v) - \nu(t)\right) - \sum_{i=1}^{N} \alpha^i(x,v)\xi^i(v) \\ &+ \sum_{i=1}^{N} \left[\hat{\beta}^i(x,v) - \beta^i(x,v)\right] \xi^i(v)u_{ce} - \sum_{i=1}^{N} \left[\beta_k^i(t) + \beta^i(x,v)\right] \xi^i(v)u_{si} \\ =& -\eta e_s + \sum_{i=1}^{N} \left[\hat{\alpha}^i(x,v) - \alpha^i(x,v)\right] \xi^i(v) + \sum_{i=1}^{N} \left[\hat{\beta}^i(x,v) - \beta^i(x,v)\right] \xi^i(v)u_{ce} \\ &- \sum_{i=1}^{N} \left[\beta_k^i(t) + \beta^i(x,v)\right] \xi^i(v)u_{si}. \end{aligned}$$

V. Direct Adaptive Control

In addition to the assumptions we made in the indirect adaptive control case, we require $\beta_k^i(t) = \alpha_k^i(t) = 0$ for all $t \geq 0$ and that there exist positive constants β_0^i and β_1^i such that $0 < \beta_0^i \leq \beta^i(x,v) \leq \beta_1^i$ for $i = 1, 2, \ldots, N$. Also, we assume that we can specify some function $B^i(x) \geq 0$ such that

$$\left| \frac{d\beta^i(x,v)\xi^i(v)}{dt} \right| \leq B^i(x)\xi^i(v) \tag{37}$$

for all x and v. We know that there exists some ideal controller

$$u^* = \frac{1}{\sum_{i=1}^{N} \beta^i(x,v)\xi^i(v)} \left(-\sum_{i=1}^{N} \alpha^i(x,v)\xi^i(v) + \nu(t) \right) \tag{38}$$

where $\nu(t)$ is defined the same as that in the indirect adaptive control case. Let

$$\begin{aligned} u^* &= \sum_{i=1}^{N} u^{i*}\xi^i(v) \\ &= \sum_{i=1}^{N} \Big[\theta_u^{i*\top} \phi_u^i(x,v,\nu) \\ &\qquad + u_k^i(t) + \omega_u^i(x,v,\nu) \Big] \xi^i(v) \end{aligned} \tag{39}$$

where u_k^i is a known part of the controller (e.g., one that was designed for the nominal system) and

$$\theta_u^{i*} = \arg \min_{\theta_u^i \in \Omega_u} \left(\sup_{x \in S_x, \nu \in S_\nu} \left| \theta_u^{i\top} \phi_u^i(x,v,\nu) - \left(u^{i*} - u_k^i \right) \right| \right) \tag{40}$$

so that $\omega_u^i(x,v,\nu)$ is the approximation error. We assume that $|\omega_u^i(x,v,\nu)| \leq W_u^i(x,\nu)$, where $W_u^i(x,\nu)$ is a known bound on the error in representing the ideal controller. The approximation of the ideal controller can be represented by

$$\hat{u} = \sum_{i=1}^{N} \left[\theta_u^{i\top} \phi_u^i(x,v,\nu) + u_k^i(t) \right] \xi^i(v) \tag{41}$$

where the parameter vector $\theta_u^i(t)$ is updated online and the parameter error is

$$\tilde{\theta}_u^i(t) = \theta_u^i(t) - \theta_u^{i*}. \tag{42}$$

Consider the direct adaptive control law

$$u = \hat{u} + u_{sd} \tag{43}$$

which is the sum of the approximation to the ideal control law $\hat{u}$ and a sliding-mode control term

$$u_{sd} = \left(\frac{\sum_{i=1}^{N} B^i(x)\xi^i(v)\,|e_s|}{2\left[\sum_{i=1}^{N} \beta_0^i \xi^i(v) \right]^2} + \sum_{i=1}^{N} W_u^i(x,\nu)\xi^i(v) \right) sgn\,(e_s) \tag{44}$$

and we use the update law

$$\dot{\theta}_u^i(t) = {Q_u^i}^{-1} \phi_u^i(x,v,\nu) e_s \xi^i(v) \tag{45}$$

where Q_u^i is positive definite and diagonal and $\xi^i(v)$ represent the effects of interpolated adaptive laws. We also use a projection method to ensure that $\theta_u^i \in \Omega_u$.

Theorem 2: Consider the nonlinear system (9) and (10) with strong relative degree d. Assume that 1) $0 < \beta_0^i \leq \beta^i(x,v) \leq \beta_1^i$ for some known positive constants β_0^i and β_1^i, 2) (37) holds for some known function $B^i(x) \geq 0$, 3) $|\omega_u^i(x,v,\nu)| \leq W_u^i(x,\nu)$ with known $W_u^i(x,\nu)$, 4) $y_m(t), \dot{y}_m(t), \ldots, y_m^{(d)}(t)$ are measurable and bounded, 5) $x(t), y(t), \dot{y}(t), \ldots, y^{(d-1)}(t)$ are measurable, and 6) $1 \leq d < n$ with the zero dynamics exponentially attractive or $d = n$. Under these conditions there exist direct adaptive control laws (43), (41), and (44), and the update law (45) such that all internal signals are bounded and the tracking error e is asymptotically stable.

Proof: Analogous to [20], consider the following Lyapunov function candidate

$$V_d = \frac{1}{2\sum_{i=1}^{N} \beta^i(x,v)\xi^i(v)} e_s^2 + \frac{1}{2} \sum_{i=1}^{N} \tilde{\theta}_u^{i\top} Q_u^i \tilde{\theta}_u^i. \tag{46}$$

Take the time derivative

$$\dot{V}_d = \frac{e_s}{\sum_{i=1}^{N} \beta^i(x,v)\xi^i(v)} \dot{e}_s - \frac{\sum_{i=1}^{N} \frac{d\beta^i(x,v)\xi^i(v)}{dt} e_s^2}{2\left[\sum_{i=1}^{N} \beta^i(x,v)\xi^i(v) \right]^2} + \sum_{i=1}^{N} \tilde{\theta}_u^{i\top} Q_u^i \dot{\tilde{\theta}}_u^i \tag{47}$$

Note that $\bar{e}_s(t) = \dot{e}_s(t) - e^{(d)}(t)$ and the dth derivative of the output error is $e^{(d)} = y_m^{(d)} - y^{(d)}$ so that

$$\dot{e}_s(t) = \bar{e}_s(t) + y_m^{(d)} - y^{(d)}$$

and from (13), (29), (43), and (38), and by noting that $\alpha_k^i(t) = \beta_k^i(t) = 0$, the first equation at the bottom of the next page holds true. Also, from (41), (39), and (42), we have

$$\begin{aligned} \dot{e}_s(t) = &-\eta e_s - \sum_{i=1}^{N} \beta^i(x,v)\xi^i(v) \\ &\cdot \sum_{j=1}^{N} \left[\tilde{\theta}_u^{j\top} \phi_u^j(x,v,\nu) - \omega_u^j(x,v,\nu) \right] \xi^j(v) \\ &- \sum_{i=1}^{N} \beta^i(x,v)\xi^i(v) u_{sd}. \end{aligned}$$

Substitute this equation into (47), and substitute (45) into (47) and note that $\dot{\tilde{\theta}}_u^i = \dot{\theta}_u^i$, then we get the second equation shown at the bottom of the next page. After we consider the projection modification to the update law, we have

$$\begin{aligned} \dot{V}_d \leq &-\frac{\eta e_s^2}{\sum_{i=1}^{N} \beta^i(x,v)\xi^i(v)} \\ &- \left(\frac{\sum_{i=1}^{N} \frac{d\beta^i(x,v)\xi^i(v)}{dt} e_s}{2\left[\sum_{i=1}^{N} \beta^i(x,v)\xi^i(v) \right]^2} - \sum_{i=1}^{N} \omega_u^i(x,v,\nu)\xi^i(v) \right) e_s - e_s u_{sd}. \end{aligned}$$

Substitute (44) into the previous equation, and note that

$$
\begin{aligned}
&-\left(\frac{\sum_{i=1}^{N}\frac{d\beta^i(x,v)\xi^i(v)}{dt}e_s}{2\left[\sum_{i=1}^{N}\beta^i(x,v)\xi^i(v)\right]^2}-\sum_{i=1}^{N}\omega_u^i(x,v,\nu)\xi^i(v)\right)e_s\\
&\le\left(\frac{\sum_{i=1}^{N}\left|\frac{d\beta^i(x,v)\xi^i(v)}{dt}\right||e_s|}{2\left[\sum_{i=1}^{N}\beta^i(x,v)\xi^i(v)\right]^2}+\sum_{i=1}^{N}\left|\omega_u^i(x,v,\nu)\right|\xi^i(v)\right)|e_s|\\
&\le\left(\frac{\sum_{i=1}^{N}B^i(x)\xi^i(v)\,|e_s|}{2\left[\sum_{i=1}^{N}\beta_0^i\xi^i(v)\right]^2}+\sum_{i=1}^{N}W_u^i(x,\nu)\xi^i(v)\right)|e_s|
\end{aligned}
$$

since $\xi^i(v)>0$ and note that $0<\beta_0^i\le\beta^i(x,v)\le\beta_1^i$, so that we have

$$\dot{V}_d\le-\frac{\eta e_s^2}{\sum_{i=1}^{N}\beta^i(x,v)\xi^i(v)}. \tag{48}$$

Therefore, V_d is a nonincreasing function of time. This gives the same type of stability result that we obtained in the indirect case. □

Remark: Note that most of the papers [11]–[19] deal with indirect adaptive control, whereas very few authors (e.g., [20] and [21]) face the direct approach, because it is not always clear how to construct the control law without knowledge of the system dynamics. Here, we design the direct adaptive control law based on the feedback linearizing law [20], and then generalize it to the interpolated systems. Asymptotic stability of the output has also been obtained without assumptions on rate of change of system dynamics. Compared to indirect adaptive control, direct adaptive control usually shows better transient behavior because it may learn and adapt faster (probably due to the fact that it has less parameters to be tuned).

VI. SIMULATION EXAMPLES: JET ENGINE CONTROL

To study the effectiveness of the proposed adaptive control methods, we apply them to the component level engine cycle model (CLM) of an aircraft jet engine (General Electric XTE46), which is a simplified, unclassified version of the

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$$
\begin{aligned}
\dot{e}_s(t)=&\bar{e}_s(t)+(\nu(t)-\eta e_s-\bar{e}_s)\\
&-\left(\sum_{i=1}^{N}\alpha^i(x,v)\xi^i(v)+\sum_{i=1}^{N}\beta^i(x,v)\xi^i(v)\left(\hat{u}+u_{sd}\right)\right)\\
=&-\eta e_s+\left(\nu(t)-\sum_{i=1}^{N}\alpha^i(x,v)\xi^i(v)-\sum_{i=1}^{N}\beta^i(x,v)\xi^i(v)u^*\right)\\
&-\sum_{i=1}^{N}\beta^i(x,v)\xi^i(v)\left(\hat{u}-u^*\right)-\sum_{i=1}^{N}\beta^i(x,v)\xi^i(v)u_{sd}\\
=&-\eta e_s-\sum_{i=1}^{N}\beta^i(x,v)\xi^i(v)\left(\hat{u}-u^*\right)-\sum_{i=1}^{N}\beta^i(x,v)\xi^i(v)u_{sd}.
\end{aligned}
$$

$$
\begin{aligned}
\dot{V}_d=&\frac{e_s}{\sum_{i=1}^{N}\beta^i(x,v)\xi^i(v)}\\
&\left(-\eta e_s-\sum_{i=1}^{N}\beta^i(x,v)\xi^i(v)\sum_{j=1}^{N}\left[\tilde{\theta}_u^{j\top}\phi_u^j(x,v,\nu)-\omega_u^j(x,v,\nu)\right]\xi^j(v)-\sum_{i=1}^{N}\beta^i(x,v)\xi^i(v)u_{sd}\right)\\
&-\frac{\sum_{i=1}^{N}\frac{d\beta^i(x,v)\xi^i(v)}{dt}e_s^2}{2\left[\sum_{i=1}^{N}\beta^i(x,v)\xi^i(v)\right]^2}+\sum_{i=1}^{N}\tilde{\theta}_u^{i\top}Q_u^i\left[{Q_u^i}^{-1}\phi_u^i(x,v,\nu)e_s\xi^i(v)\right]\\
=&-\frac{\eta e_s^2}{\sum_{i=1}^{N}\beta^i(x,v)\xi^i(v)}-\left(\frac{\sum_{i=1}^{N}\frac{d\beta^i(x,v)\xi^i(v)}{dt}e_s}{2\left[\sum_{i=1}^{N}\beta^i(x,v)\xi^i(v)\right]^2}-\sum_{i=1}^{N}\omega_u^i(x,v,\nu)\xi^i(v)\right)e_s-e_su_{sd}.
\end{aligned}
$$

original "integrated high performance turbine engine technology" (IHPTET) engine [33]. The CLM of the XTE46 is a thermodynamic simulation package developed by General Electric Aircraft Engines (GEAE). This is a sophisticated highly nonlinear dynamic model where each engine component is simulated. The reason that GEAE developed such a "paper engine" was in order to facilitate the design and analysis of engine control systems before installing them for actual flights. Thus, the component level model is quite complicated and so accurate that it can be used as the "truth model" in the control simulation to represent the real engine.

In order to apply the proposed adaptive control strategy to the jet engine, we start from developing a "design model." The CLM for the XTE46 aircraft engine is a multiple-input–multiple-output nonlinear system (involving schedules, look-up tables and partial differential equations). However, GEAE (the authority on this engine) indicates that the key single-input–single-output (SISO) loop (i.e., fuel flow to fan speed loop) is not tightly coupled with other loops. Therefore, to focus our theoretical studies, we could assume that the fundamental engine dynamic characteristics of interest are represented by a SISO, combustor fuel flow to fan rotor speed system (while the other two input variables, the exhaust nozzle area and the variable area bypass injector area, could be properly scheduled as functions of the power level and the inlet temperature). To develop the design model for the XTE46 engine, we conduct nonlinear system identification to approximate local engine dynamics. Based on the transient data generated by the CLM, a number of local nonlinear models are constructed, each of which is in the structure of Takagi–Sugeno fuzzy systems and is corresponding to a specific operating condition and engine health situation. In order to build a "global" engine model (actually, it is a "regional" model valid in the "climb" region), we conduct nonlinear interpolation among a grid of these local models. The global engine model can be viewed to have a hierarchical learning structure, where we perform local learning to approximate the local engine dynamics and interpolate these local models to generate the global model. Note that the nonlinearity of the engine is different at different operating conditions and for different engine health situations. Moreover, the operating condition of the engine is defined by four variables: the altitude (ALT), the mach number (XM), the difference of temperature (DTAMB), and the throttle setting represented by power code (PC). The health of the engine is described by ten quality parameters including the flows and efficiencies of the fan (ZSW2 and SEDM2), the compressor (ZSW7D, SEDM7D, ZSW27, and SEDM27) and turbines (ZSW41, ZSE41, ZSW49, and ZSE49). Therefore, although it is theoretical possible to approximate the engine dynamics by building one fuzzy system, it is not feasible in practice because of the huge amounts of data. Furthermore, the advantage of taking this hierarchical model form is that the engine dynamics can be separated into an unknown but slowly time-varying part (the local engine dynamics) and a known but fast time-varying part (the operating condition). This facilitates the application of the interpolated adaptive controller to handle the fast time-varying dynamics caused by the rapid change in the operating conditions.

The general form of the model can be described as shown in (49)–(54) at the bottom of the page, where $u = \mathrm{WF36}$ is the system input (the combustor fuel flow) and $x = [x_1, x_2]^\top = [\mathrm{XNL}, \mathrm{XNH}]^\top$ represents the system states (the fan rotor speed

$$\dot{x} = f(x,c,p) + g(x,c,p)u \tag{49}$$

$$y = x_1 \tag{50}$$

and

$$f(x,c,p) = \frac{\sum_{i=1}^{N} \underline{f}(x,c_i,p_i)\,\mu_i(c,p)}{\sum_{i=1}^{N} \mu_i(c,p)} \tag{51}$$

$$g(x,c,p) = \frac{\sum_{i=1}^{N} \underline{g}(x,c_i,p_i)\,\mu_i(c,p)}{\sum_{i=1}^{N} \mu_i(c,p)} \tag{52}$$

$$\underline{f}(x,c_i,p_i) = \frac{\sum_{j=1}^{R} \left[a_{j,0}(c_i,p_i) + a_{j,1}(c_i,p_i)\,x_1 + a_{j,2}(c_i,p_i)\,x_2\right] \underline{\mu}_j(x_1)}{\sum_{j=1}^{R} \underline{\mu}_j(x_1)} \tag{53}$$

$$\underline{g}(x,c_i,p_i) = \frac{\sum_{j=1}^{R} a_{j,3}(c_i,p_i)\,\underline{\mu}_j(x_1)}{\sum_{j=1}^{R} \underline{\mu}_j(x_1)} \tag{54}$$

and core rotor speed), which are positive since the speed cannot be negative and $x \in S_x$ (a valid speed region). The vector $c = [\mathrm{ALT}, \mathrm{XM}, \mathrm{DTAMB}, \mathrm{PC}]^\top$ represents the known operating condition of the engine and

$$p = [\mathrm{ZSW2}, \mathrm{SEDM2}, \mathrm{ZSW7D}, \mathrm{SEDM7D}, \mathrm{ZSW27}, \mathrm{SEDM27}, \mathrm{ZSW41}, \mathrm{ZSE41}, \mathrm{ZSW49}, \mathrm{ZSE49}]^\top$$

represents the unknown quality parameter vector. The values of c_i and p_i specify the nodes where we establish the local models. The functions $f = [f_1, f_2]^\top$ and $g = [g_1, g_2]^\top$ are 2×1 function vectors obtained through fuzzy interpolation and $\mu_i(c,p)$ are interpolating membership functions. The functions $\underline{f} = [\underline{f}_1, \underline{f}_2]^\top$ and $\underline{g} = [\underline{g}_1, \underline{g}_2]^\top$ are 2×1 function vectors obtained through nonlinear system identification and are in the form of Takagi–Sugeno fuzzy systems, where $a_{j,0}$, $a_{j,1}$, $a_{j,2}$ and $a_{j,3}$ are 2×1 parameter vectors of the (linear) consequent functions and $\underline{\mu}_j(x_1)$ are membership functions describing local nonlinearity with respect to x_1. (Refer to [32] and [34] for more details on how we have developed the nonlinear engine model using Takagi–Sugeno fuzzy systems).

By inspecting the parameters that result from the identification process, we found that $a^1_{j,3}(c_i,p_i) > a^2_{j,3}(c_i,p_i) > 0$ and $a^2_{j,2}(c_i,p_i) < a^1_{j,2}(c_i,p_i) < 0$ for any $i = 1, 2, \ldots, N$, $j = 1, 2, \ldots, R$. Basically, these sign conditions explain some physical dynamics of the engine. In particular, the relationships among the state variables and the input variable are relevant for stability analysis of the system. For instance, we have both $a^1_{j,3}(c_i,p_i) > 0$ and $a^2_{j,3}(c_i,p_i) > 0$, which indicate that if the fuel flow is increased, both the fan rotor speed and the core rotor speed will be increased. These constraints on the model parameters are important to design and analyze the stable adaptive control system. For example, by knowing $a^1_{j,3}(c_i,p_i) > 0$ for any operating conditions and quality parameters (and $\underline{\mu}_j(x_1) > 0$ and $\sum_{j=1}^{R} \underline{\mu}_j(x_1) \neq 0$ by the definition of Takagi–Sugeno fuzzy systems), we obtain $\underline{g}^1(x, c_i, p_i) > 0$ and thus $g_1(x,c,p) > 0$ for all x, c, p. This implies the "relative degree" of the engine is one. In addition, more details on how to use these relationships to determine the exponentially attractive zero dynamics of the engine can be found in the stability analysis part of [32]. Finally, note that via similar nonlinear identification studies we showed that an interpolation of strict feedback form models could not adequately represent the engine dynamics.

We develop an adaptive controller for the engine control problem using the indirect method. We choose to implement the indirect controller because we could facilitate the indirect adaptive controller by using our developed models to represent nominal system dynamics in different regions. Consider the engine in the form of

$$\dot{y} = f_1(x,c,p) + g_1(x,c,p)u \tag{55}$$

$$= \sum_{i=1}^{N} \left(\alpha^i_k(c,p_0,t) + \alpha^i(x,c,p)\right)\xi^i(c) + \sum_{i=1}^{N} \left(\beta^i_k(c,p_0,t) + \beta^i(x,c,p)\right)\xi^i(c)u \tag{56}$$

where $x(t)$ and $y(t)$ are measurable according to the properties of the component level engine model. By studying dynamics of the developed nonlinear model, we know that $g_1(x,c,p) > 0.32$ so that we can set $\beta_0 = 0.32$. The nonlinear system dynamics are separated into $N = 9$ local regions according to the operating condition variables which are measurable. We use our developed engine model to represent the local nominal model dynamics $\alpha^i_k(c,p_0,t)$ and $\beta^i_k(c,p_0,t)$ by setting the quality parameters to be the nominal value p_0 and they are bounded if x is bounded since the model is in the form of a Takagi–Sugeno fuzzy system. The unknown dynamics $\alpha^i(x,c,p)$ and $\beta^i(x,c,p)$ describe the model uncertainties caused by nominal model inaccuracy and system changes (time-varying characteristics). They are approximated by 18 radial basis function networks (nine for $\hat{\alpha}^i$ and nine for $\hat{\beta}^i$). Each radial basis network has eleven receptive field units. The inputs to the neural networks include two state variables and the parameters are updated online to capture the unknown system dynamics. Note that the stable adaptive controller will ensure the stability of x_1 and the uniform exponential attractivity of the engine zero dynamics will ensure the stability of the uncontrollable state x_2. Since the relative degree of the system is 1, the error dynamics are simple ($e_s(t) = e(t)$ and $\bar{e}_s(t) = 0$). The reference trajectory is defined by passing a reference signal through a linear reference model $Y_m(s)/R(s) = 3/(s+3)$ so that $y_m(t)$ and $\dot{y}_m(t)$ are measurable and bounded. Taking into account the engine dynamics, the model uncertainty is described by $W^i_\alpha = 0.005$ and $W^i_\beta = 0.005$. Note that we cannot explicitly know the model uncertainty so that the parameters W^i_α and W^i_β are treated as design parameters and tuned by trial and error to achieve good control performance. In addition, the adaptation gains are tuned to be ${Q^i_\alpha}^{-1} = 5e-8$ and ${Q^i_\beta}^{-1} = 1e-17$ and the design parameter $\eta = 1$. Generally, we start the tuning process based on the nonlinear engine model (the design model) that we have developed using nonlinear system identification techniques. We first remove the adaptation mechanism (by setting the adaptation gains to zero) and tune the approximation error bounds W^i_α and W^i_β in order to stabilize the system. At this initial step, the approximation error bounds are usually large because they quantifies the unknown system dynamics at this moment. Next, we increase the adaption gains to add the online learning ability to the adaptive controller and reduce the approximation error bounds subsequently. The aforementioned procedure is iterated until certain good control performance has been achieved and further increasing the adaptation gains may cause oscillation. Afterwards, we apply the controller to the component level model simulation of the XTE46 engine and do some further tuning. This XTE46 simulator has been developed by GEAE to be very complicated and accurate so that the simulation conducted on this simulator is very close to that on the real engine for actual flights.

To assess the performance of the adaptive controller we let the component level engine model run at a scenario where the altitude (ALT) varies from 12 500 to 17 500 (i.e., ALT $= 12\,500$ for first 2 sec; then the altitude is linearly increased from 12 500

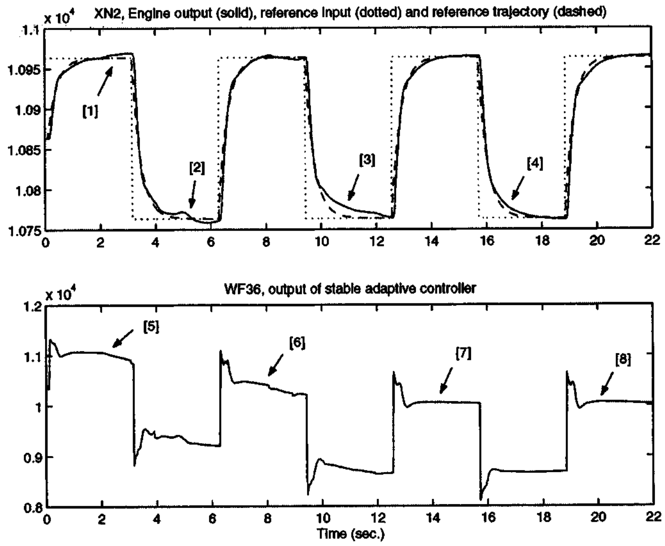

Fig. 1. Performance of the interpolated adaptive controller.

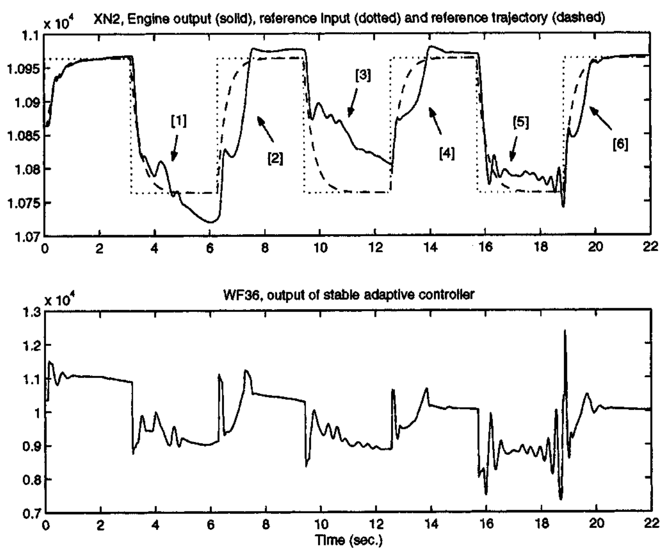

Fig. 2. Performance of the single adaptive controller.

to 17 500 for 10 sec; after that, the altitude keeps constant at 17 500) and other operating conditions are kept constant, that is, XM $= 0.7$, DTAMB $= 0$ and PC $= 46$. The engine quality parameters are also kept constant. (Actually, we first applied the controller to the design model, but in the interest of brevity, here, we only show the results on the component level model simulation, which we treat as the real application on the XTE46 engine.) As shown in Fig. 1, although the altitude changes significantly during the time period ($T = 2 \sim 12$ sec.), the performance does not deteriorate too much (as indicated by arrows 1, 2, and 3) and improves after the altitude variation (as indicated by arrow 4) by applying the adaptation scheme to compensate for model uncertainties. The output of the adaptive controller also indicates the smooth changes of control laws (as pointed by arrows 5, 6, 7, and 8).

The effectiveness of the proposed "interpolated" adaptive controller can be further demonstrated by comparing its performance with that of a "single" adaptive controller (where the adaptive control law is not in the interpolated form but is driven by one pair of online approximators, that is $N = 1$ in (27) and (28)) [31]. As shown in Fig. 2, the fast time-varying nature of altitude change makes it difficult for the online approximators to adapt fast enough, which results in deteriorated performance (as indicated by arrows 1, 2, and 3), even though the adaptation scheme does have apparent effects when system dynamics are not fast time-varying (as indicated by arrows 4, 5, and 6).

Clearly, the interpolated strategy introduced in this paper (Fig. 1) performs much better than the single adaptive controller (Fig. 2). Essentially, it provides a method to exploit the structure inherent in the class of models that we consider in the paper. Since this class is one that is the product of known nonlinear identification procedures, the methodology presented here provides a particularly practical approach to control a class of nonlinear time-varying systems.

VII. Conclusion

In this paper, we have proposed an online approximation-based adaptive control methodology for a class of nonlinear systems with a time-varying structure. This class of systems is composed of interpolations of nonlinear subsystems which are input–output feedback linearizable. Without assumptions on rate of change of system dynamics, stable indirect and direct adaptive control methods were presented with analysis of stability for all signals in the closed-loop as well as asymptotic tracking. The performance of the adaptive controller was demonstrated using a jet engine control problem.

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