VIII. Conclusions

The adaptive schemes proposed in this paper advance the state-of-the-art of adaptive nonlinear output-feedback control in several directions. They remove the main drawbacks of the original Marino–Tomei design. Only the minimal number of parameters is updated, and any standard update law can be incorporated in the swapping-based scheme. The estimation-based approach can now be used for adaptive nonlinear output-feedback control without any growth restrictions. The modifications made in the Marino–Tomei controller make it possible to systematically improve the transient performance by increasing certain design parameters.

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Adaptive Control of a Class of Decentralized Nonlinear Systems

Jeffrey T. Spooner and Kevin M. Passino

***Abstract*—Within this brief paper, a stable indirect adaptive controller is presented for a class of interconnected nonlinear systems. The feedback and adaptation mechanisms for each subsystem depend only upon local measurements to provide asymptotic tracking of a reference trajectory. In addition, each subsystem is able to adaptively compensate for disturbances and interconnections with unknown bounds. The adaptive scheme is illustrated through the longitudinal control of a string of vehicles within an automated highway system (AHS).**

I. Introduction

Decentralized control systems often arise from either the physical inability for subsystem information exchange or the lack of computing capabilities required for a single central controller. Furthermore, difficulty and uncertainty in measuring parameter values within a large-scale system may call for adaptive techniques. Since these restrictions encompass a large group of applications, a variety of decentralized adaptive techniques have been developed. Model reference adaptive control (MRAC)-based designs for decentralized systems have been studied in [1]–[4] for the continuous time case and in [5] and [6] for the discrete time case. These approaches, however, are limited to decentralized systems with linear subsystems and possibly nonlinear interconnections. Decentralized adaptive controllers for robotic manipulators were presented in [7]–[9], while a scheme for nonlinear subsystems with a special class of interconnections was presented in [10].

Our objective is to present adaptive controllers for a class of decentralized systems with nonlinear subsystems, unknown nonlinear interconnections, and disturbances with unknown bounds. This paper is organized as follows: In Section II, the details of the problem statement for the decentralized system are presented. The adaptive algorithms for each subsystem using only local information are presented, and composite system stability is established in Section III. An illustrative example is then used in Section IV to demonstrate the effectiveness of the decentralized adaptive technique.

II. Problem Statement

Our objective is to design an adaptive control system for each subsystem which will cause the output, y_{p_i}, of a relative degree r_i subsystem, S_i, to track a desired output trajectory, y_{m_i}, in the presence of interconnections, I_{ij}, and unknown disturbances using only local measurements (see Fig. 1). The desired output trajectory, y_{m_i}, may be defined by a signal external to the control system so that the first r_i derivatives of the ith subsystem's reference signal y_{m_i} may be measured or by a reference model with relative degree greater than or equal to r_i which characterizes the desired performance. It is thus assumed that the desired output trajectory and its derivatives $y_{m_i}, \cdots, y_{m_i}^{(r_i)}$ for the ith subsystem, S_i, are measurable and bounded (let $y_{m_i}^{(r_i)}$ denote the r_ith derivative of y_{m_i} with respect to time). Within this paper an "output error indirect adaptive controller" is

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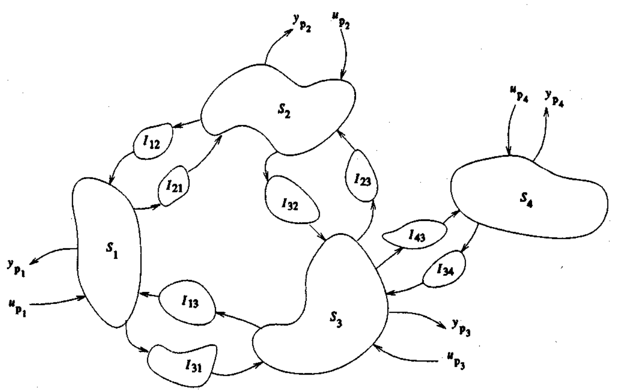

Fig. 1. A decentralized network.

used (using the terminology from [11]), where an identifier seeks to approximate the subsystem dynamics and use this to tune the parameters of a controller so that y_{p_i} follows y_{m_i}, and hence the tracking errors, $e_{o_i} = y_{m_i} - y_{p_i}$, are driven to zero.

Here we consider each subsystem to be single-input–single-output such that

$$\dot{X}_i = f_i(t, X_1, \cdots, X_m) + g_i(X_i)u_{p_i} \tag{1}$$

$$y_{p_i} = h_i(t, X_1, \cdots, X_m) \tag{2}$$

where $X_i \in \Re^{n_i}$ is the state vector, $u_{p_i} \in \Re$ is the input, and $y_{p_i} \in \Re$ is the output of the plant for the ith subsystem, S_i, and the functions $f_i(t, X_1, \cdots, X_m), g_i(X) \in \Re^{n_i}$ and $h_i(t, X_1, \cdots, X_m) \in \Re$, $i = 1, \cdots, m$ are smooth. If each subsystem has "strong relative degree" r_i, then

$$\begin{aligned}
\dot{\xi}_{(1)_i} &= \xi_{2_i} = L_{f_i} h_i(t, X_1, \cdots, X_m) \\
&\;\vdots \\
\dot{\xi}_{(r_i-1)_i} &= \xi_{r_i} = L_{f_i}^{r_i-1} h_i(t, X_1, \cdots, X_m) \\
\dot{\xi}_{(r_i)_i} &= L_{f_i}^{r_i} h_i(t, X_1, \cdots, X_m) \\
&\quad + L_{g_i} L_{f_i}^{r_i-1} h_i(t, X_1, \cdots, X_m) u_{p_i} \\
&\quad + \Delta_i(t, X_1, \cdots, X_m)
\end{aligned} \tag{3}$$

with $\xi_{(1)_i} = y_{p_i}$, which may be rewritten as

$$\begin{aligned}
y_{p_i}^{(r_i)} &= (\alpha_{k_i}(t) + \alpha_i(X_i)) + (\beta_{k_i}(t) + \beta_i(X_i))u_{p_i} \\
&\quad + \Delta_i(t, X_1, \cdots, X_m)
\end{aligned} \tag{4}$$

where $L_{g_i} h_i(t, X_1, \cdots, X_m)$ is the Lie derivative of $h_i\ (t, X_1, \cdots, X_m)$ with respect to $g_i(L_{g_i} h_i(X) = \frac{\partial h_i}{\partial X} g_i(X)$, e.g., $L_g^2 h(X) = L_g(L_g h(X)))$, and it is assumed that for some $\beta_{0_i} > 0$, we have $|\beta_{k_i}(t) + \beta_i(X_i)| \geq \beta_{0_i}$, so that it is bounded away from zero (for convenience we assume that $\beta_{k_i}(t) + \beta_i(X_i) > 0$; however, the following analysis may easily be modified for subsystems which are defined with $\beta_{k_i}(t) + \beta_i(X_i) < 0$). The effects of the interconnections, $I_{i,j}$, upon the subsystem S_i are accounted for within the term $\Delta_i(t, X_1, \cdots, X_m)$. We will assume that $\alpha_{k_i}(t)$ and $\beta_{k_i}(t)$ are known components of the dynamics of the ith subsystem, S_i, or time dependent signals, and that $\alpha_i(X_i)$ and $\beta_i(X_i)$ represent nonlinear dynamics of the subsystem that have unknown parameters. Throughout the analysis to follow, both $\alpha_k(t)$ and $\beta_k(t)$ may be set to zero. It is also assumed that the zero dynamics for each subsystem are exponentially attractive [12].

III. Decentralized Adaptive Controller

Assume that each of the functions $\alpha_i(X_i)$ and $\beta_i(X_i)$ may be expressed as a linear combination of known nonlinear functions

$$\alpha_i(X_i) = A_{\alpha_i}^{*T} \zeta_{\alpha_i} \tag{5}$$

$$\beta_i(X_i) = A_{\beta_i}^{*T} \zeta_{\beta_i} \tag{6}$$

where $\zeta_{\alpha_i}: \Re^{n_i} \to \Re^{p_i}$ and $\zeta_{\beta_i}: \Re^{n_i} \to \Re^{q_i}$, while the constant vectors $A_{\alpha_i}^* \in \Re^{p_i}$ and $A_{\beta_i}^* \in \Re^{q_i}$ represent the nominal coefficients for the ith subsystem. We require that a set of "basis functions," ζ_{α_i} and ζ_{β_i}, which may be used to exactly represent $\alpha_i(X_i)$ and $\beta_i(X_i)$, be known; however, the corresponding coefficients need not be known. For example, if $\alpha_1 = x_1 + 1.5x_1x_2^2$ and $\beta_1 = 2.5 + \sin(x_2)$, then we may let $\zeta_{\alpha_1} = [x_1, x_1x_2^2]^T, A_{\alpha_1}^* = [1, 1.5]^T, \zeta_{\beta_1} = [1, \sin(x_2)]^T$, and $A_{\beta_1}^* = [2.5, 1]^T$. (In Section IV, we show how to choose these for an automated highway system (AHS) application.)

The output error for the ith subsystem is $e_{o_i} := y_{m_i} - y_{p_i}$. It is desired that the tracking error follow

$$e_{o_i}^{(r_i)} + k_{i,r_i-1} e_{o_i}^{(r_i-1)} + \cdots + k_{i,0} e_{o_i} = 0 \tag{7}$$

where the coefficients are picked so that each $\hat{L}_i(s) = s^{(r_i)} + k_{i,r_i-1} s^{(r_i-1)} + \cdots + k_{i,0}$ has its roots in the open, left half plane (is Hurwitz).

An adaptive algorithm is used to estimate $A_{\alpha_i}^*$ and $A_{\beta_i}^*$ with A_{α_i} and A_{β_i}, respectively. The estimates are used to define $\hat{\alpha}_i := A_{\alpha_i}^T \zeta_{\alpha_i}$ and $\hat{\beta}_i := A_{\beta_i}^T \zeta_{\beta_i}$. Parameter error vectors are defined as $\Phi_{\alpha_i} := A_{\alpha_i} - A_{\alpha_i}^*$ and $\Phi_{\beta_i} := A_{\beta_i} - A_{\beta_i}^*$. Using the current estimate for the ith subsystem with no interconnections, a "certainty equivalence" control term [12] for the ith subsystem is given as

$$u_{c_i} = \frac{1}{\beta_{k_i}(t) + \hat{\beta}_i(X_i)} [-(\alpha_{k_i}(t) + \hat{\alpha}_i(X_i)) + \nu_i] \tag{8}$$

assuming that $\beta_{k_i}(t) + \hat{\beta}_i(X_i)$ is bounded away from zero. Let $\nu_i = y_{m_i}^{(r_i)} + k_{i,r_i-1} e_{o_i}^{(r_i-1)} + \cdots + k_{i,0} e_{o_i} + c_i(t)\operatorname{sgn}(w_i(t)) + \eta_i(t)w_i(t)/2$, where the signals $w_i(t), c_i(t)$, and $\eta_i(t)$ are yet to be defined (below we will drop the time index). The term $c_i \operatorname{sgn}(w_i(t))$, ($\operatorname{sgn}(x) = 1$ if $x > 0, -1$ if $x < 0$) is used to reject unknown disturbances, while the term $\eta_i w_i(t)/2$ is used to compensate for unknown effects from the interconnections. With $u_{p_i} = u_{c_i}$, the output error dynamics may be expressed as

$$\begin{aligned}
e_{o_i}^{(r_i)} &= (\hat{\alpha}_i(X_i) - \alpha_i(X_i)) + (\hat{\beta}_i(X_i) - \beta_i(X_i))u_{c_i} \\
&\quad - k_{i,r_i-1} e_{o_i}^{(r_i-1)} - \cdots - k_{i,0} e_{o_i} - \Delta_i(t, X_1, \cdots, X_m) \\
&\quad - c_i \operatorname{sgn}(w_i(t)) - \eta_i w_i(t)/2.
\end{aligned} \tag{9}$$

It should be noted that the certainty equivalence term, u_{c_i}, is dependent upon the first $r_i - 1$ derivatives of the subsystem output so it may not be suitable for subsystems with a high relative degree. Defining the output error vector for the ith subsystem as $e_i = [e_{o_i}, \cdots, e_{o_i}^{(r_i-1)}]$, the error dynamics may be expressed as

$$\begin{aligned}
\dot{e}_i = \Lambda_i e_i + b_i[&(\hat{\alpha}_i(X_i) - \alpha_i(X_i)) \\
&+ (\hat{\beta}_i(X_i) - \beta_i(X_i))u_{c_i} - \Delta_i(t, X_1, \cdots, X_m) \\
&- c_i \operatorname{sgn}(w_i(t)) - \eta_i w_i(t)/2]
\end{aligned} \tag{10}$$

where

$$\Lambda_i = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -k_{i,0} & -k_{i,1} & -k_{i,2} & \cdots & -k_{i,r_i-1} \end{bmatrix} \tag{11}$$

and $b_i = [0\,0\,\cdots\,0\,1]^T \in \Re^{r_i}$. It is assumed that the interconnections satisfy

$$\Delta_i(t, X_1, \cdots, X_m) = d_i(t) + \delta_i(t, X_1, \cdots, X_m) \tag{12}$$

where $|\delta_i(t, X_1, \cdots, X_m)| \leq \sum_{j=1}^m \gamma_{i,j}\|e_j\|_2$ ($\|\cdot\|_2$ is the Euclidean vector norm). The scalars $\gamma_{i,j}$ quantify the strength of the interconnections. Let $c_i^* = (\sup[d_i(t)] - \inf[d_i(t)])/2$ be a measure of the variation of $d_i(t)$ and $\bar{d}_i = (\sup[d_i(t)] + \inf[d_i(t)])/2$ be a measure of the center position of $d_i(t)$. We may thus let $d_i(t) = \bar{d}_i + \tilde{d}_i(t)$, where $|\tilde{d}_i| \leq c_i^*$, with c_i^* assumed to be bounded. Also, if $\bar{d}_i$ is nonzero, we may absorb $\bar{d}_i$ into $\alpha_i(X_i)$ within (4) as an unknown constant. Within the adaptation algorithm, the bound c_i^* will be estimated by c_i, with the corresponding parameter error defined as $\phi_{c_i} := c_i - c_i^*$. Also define $H^* = [\eta_1^*, \cdots, \eta_m^*]$ as a vector of desired gains which shall be defined later. Each η_i will adapt to achieve these feedback gains with a parameter error $\phi_{\eta_i} = \eta_i - \eta_i^*$.

Consider the following Lyapunov type function for the ith subsystem

$$\begin{aligned} v_i = e_i^T P_i e_i + \Phi_{\alpha_i}^T Q_{\alpha_i} \Phi_{\alpha_i} + \Phi_{\beta_i}^T Q_{\beta_i} \Phi_{\beta_i} \\ + q_{c_i}\phi_{c_i}^2 + q_{\eta_i}\phi_{\eta_i}^2 \end{aligned} \tag{13}$$

where each $P_i \in \Re^{n_i \times n_i}$, $Q_{\alpha_i} \in \Re^{p_i \times p_i}$, and $Q_{\beta_i} \in \Re^{q_i \times q_i}$ are some positive definite and symmetric matrices with each $q_{c_i}, q_{\eta_i} > 0$ a constant. Taking the time derivative yields

$$\begin{aligned} \dot{v}_i = {} & e_i^T(P_i\Lambda_i + \Lambda_i^T P_i)e_i + 2\Phi_{\alpha_i}^T Q_{\alpha_i}\dot{\Phi}_{\alpha_i} \\ & + 2\Phi_{\beta_i}^T Q_{\beta_i}\dot{\Phi}_{\beta_i} + 2e_i^T P_i b_i \\ & \times [(\hat{\alpha}_i(X_i) - \alpha_i(X_i)) + (\hat{\beta}_i(X_i) - \beta_i(X_i))u_{c_i} \\ & - \Delta_i(t, X_1, \cdots, X_m) - c_i\,\mathrm{sgn}(w_i(t)) - \eta_i w_i(t)/2] \\ & + 2q_{c_i}\phi_{c_i}\dot{\phi}_{c_i} + 2q_{\eta_i}\phi_{\eta_i}\dot{\phi}_{\eta_i}. \end{aligned} \tag{14}$$

Since each Λ_i is positive definite, given some positive definite R_i, there exits a unique symmetric positive definite P_i satisfying $P_i\Lambda_i + \Lambda_i^T P_i = -R_i$, a Lyapunov matrix equation.

Consider the following update laws:

$$\dot{A}_{\alpha_i} = -Q_{\alpha_i}^{-1}\zeta_{\alpha_i} e_i^T P_i b_i \tag{15}$$

$$\dot{A}_{\beta_i} = -Q_{\beta_i}^{-1}\zeta_{\beta_i} e_i^T P_i b_i u_{c_i} \tag{16}$$

$$\dot{c}_i = \frac{1}{q_{c_i}}|e_i^T P_i b_i| \tag{17}$$

$$\dot{\eta}_i = \frac{1}{2q_{\eta_i}}(e_i^T P_i b_i)^2. \tag{18}$$

The update laws, (15) and (16), are used to estimate the dynamics of the subsystem under control, while (17) and (18) are used to stabilize the subsystem by estimating the effects of the interconnections. Both (17) and (18) increase monotonically, however, so a projection algorithm may be required to ensure that they do not become unnecessarily large. Note that while our update laws (15) and (16) for the subsystems bear some similarity to those in [12], our control laws are different.

Since $\dot{\Phi}_{\alpha_i} = \dot{A}_{\alpha_i}$, $\dot{\Phi}_{\beta_i} = \dot{A}_{\beta_i}$, $\dot{\phi}_{c_i} = \dot{c}_i$, and $\dot{\phi}_{\eta_i} = \dot{\eta}_i$, (14) may be written as

$$\begin{aligned} \dot{v}_i = {} & -e_i^T R_i e_i + 2e_i^T P_i b_i \\ & \times [-\tilde{d}_i(t) - \delta_i(t, X_1, \cdots, X_m) - c_i\,\mathrm{sgn}(w_i(t)) \\ & - \eta_i w_i(t)/2] + 2\phi_{c_i}|e_i^T P_i b_i| + \phi_{\eta_i}(e_i^T P_i b_i)^2. \end{aligned} \tag{19}$$

Choosing $w_i(t) = e_i^T P_i b_i$ results in

$$\begin{aligned} \dot{v}_i \leq {} & -e_i^T R_i e_i - \eta_i^*(e_i^T P_i b_i)^2 - 2e_i^T P_i b_i \delta_i(t, X_1, \cdots, X_m) \\ & - 2|e_i^T P_i b_i|(c_i^* - |\tilde{d}_i(t)|). \end{aligned} \tag{20}$$

It is possible to set $\eta_i = 0$ and use "M-matrix techniques" to find sufficient conditions for system stability [13]. Due to the conservativeness of the M-matrix techniques, however, the resulting composite system stability results are very restrictive for systems with relative degree $r_i > 1$. Here we complete the squares (in an analogous manner to [3]) and note that the last term in (20) is negative to obtain

$$\begin{aligned} \dot{v}_i \leq {} & -e_i^T R_i e_i - \eta_i^*\left[e_i^T P_i b_i + \frac{1}{\eta_i^*}\delta_i(t, X_1, \cdots, X_m)\right]^2 \\ & + \frac{1}{\eta_i^*}\delta_i^2(t, X_1, \cdots, X_m) \end{aligned} \tag{21}$$

so that if each $\eta_i^* > 0$, we simply obtain

$$\dot{v}_i \leq -e_i^T R_i e_i + \frac{1}{\eta_i^*}\delta_i^2(t, X_1, \cdots, X_m). \tag{22}$$

Now consider the composite system Lyapunov candidate $V = \sum_{i=1}^m \kappa_i v_i$, where each $\kappa_i > 0$. Taking the derivative of V and using (22) gives

$$\dot{V} \leq \sum_{i=1}^m \kappa_i\left[-e_i^T R_i e_i + \frac{1}{\eta_i^*}\left(\sum_{j=1}^m \gamma_{i,j}\|e_j\|_2\right)^2\right]. \tag{23}$$

Since $\sum_{j=1}^m \gamma_{i,j}\|e_j\|_2 = \Theta^T\Gamma_i$, where $\Theta := [\|e_1\|_2, \cdots, \|e_m\|_2]^T$ and $\Gamma_i = [\gamma_{i,1}, \cdots, \gamma_{i,m}]^T$, (23) may be written as

$$\dot{V} \leq \sum_{i=1}^m \kappa_i\left[-\lambda_i\|e_i\|_2^2 + \frac{1}{\eta_i^*}\Theta^T\Gamma_i\Gamma_i^T\Theta\right] \tag{24}$$

where λ_i is the real part of the eigenvalue of R_i with the minimum magnitude.

There exists some $H^* = [\eta_1^*, \cdots, \eta_m^*]$ such that (24) is negative semi-definite. To show this, let $\eta_i^* = \eta$, $i = 1, \cdots, m$ for some $\eta > 0$, define $D := \mathrm{diag}\{\kappa_1\lambda_1, \cdots, \kappa_m\lambda_m\}$ and $M = \sum_{i=1}^m \kappa_i\Gamma_i\Gamma_i^T$, so that

$$\dot{V} \leq -\Theta^T A\Theta \tag{25}$$

where $A = D - \frac{1}{\eta}M$. If $D = \mathrm{diag}\{l_1, \cdots, l_m\}$, with $0 < l_i \leq \tilde{l}, i = 1, \cdots, m$ for some $\tilde{l} \in \Re$, and given some bounded $M \in \Re^{m\times m}$, then for some sufficiently large $\eta \geq 0$, the matrix $A = D - \frac{1}{\eta}M$ is positive definite. This may be established using Gershgorin's theorem [14], since every eigenvalue λ of $A \in \Re^{m\times m}$ satisfies at least one of the inequalities $|\lambda - a_{i,i}| \leq \sum_{j=1, j\neq i}^m |a_{i,j}|$. Let $p_i = \sum_{j=1, j\neq i}^m |m_{i,j}|$, where $M = [m_{i,j}]$. Then if $l_i - m_{i,i}/\eta > p_i/\eta$, $i = 1, \cdots, m$, each of the eigenvalues will lie in the open right-half plane. Thus if $\eta > \max_i\{(p_i + m_{i,i})/l_i\}$, then $A = D - \frac{1}{\eta}M$ is positive definite. We are thus ensured that for some sufficiently large η, the matrix A will be positive definite. Now define $H^* = [\eta_1^*, \cdots, \eta_m^*]^T$ as

$$\begin{aligned} H^* = \arg\min_{\substack{H^*\in\Re^m \\ \eta_i^*>0}}\Bigg\{ & {H^*}^T H^* : A^* = D - \sum_{i=1}^m \kappa_i\Gamma_i\Gamma_i^T/(\eta_i^* - \epsilon) \\ & \text{is positive definite where } \epsilon > 0\Bigg\}. \end{aligned} \tag{26}$$

We have defined ϵ to be an arbitrary positive constant to ensure that A^* is positive definite rather than possibly positive semidefinite.

Theorem: Composite System Stability: If each subsystem is defined by (4) with the adaptive control law for each subsystem defined by (15)–(18) and the interconnections satisfying (12), then the tracking error for each subsystem will asymptotically converge to zero.

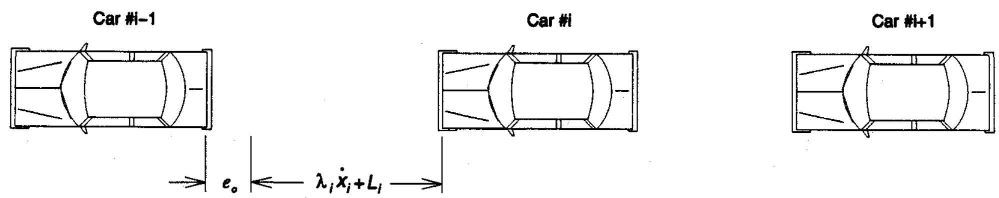

Fig. 2. Car following within an automated lane.

Proof of Theorem 1: There exists sufficiently large η such that A^*, defined by (26), is positive definite which implies that $V \in \mathcal{L}_\infty$, and thus $\Theta \in \mathcal{L}_\infty^m$. Also

$$\int_0^\infty \Theta^T A^* \Theta dt \leq -\int_0^\infty \dot{V} dt + \text{const} \tag{27}$$

so that $\Theta \in \mathcal{L}_2^m$. Since all of the signals are well defined, we also have $\dot{e}_i \in \mathcal{L}_\infty^{r_i}$ so that $\frac{d}{dt}\|e_i\|_2 = e_i^T \dot{e}_i / \|e_i\|_2 \leq \|\dot{e}\|_2 \in \mathcal{L}_\infty$. Using Barbalat's lemma, we thus establish that $\lim_{t\to\infty} \Theta = 0 \in \Re^m$, thus we are guaranteed asymptotically stable tracking for each of the subsystems. In addition, since the reference signals for each subsystem are assumed to be bounded and each subsystem has exponentially attractive zero dynamics, the states are bounded. □

Remark 1: The gains of the local controllers are adaptively increased to compensate for the effects of the interconnections. If the interconnection strengths are large, then we may expect each feedback gain, η_i, to increase to a large value. This approach allows for the adaptive routine to automatically compensate for the interconnections rather than requiring prior knowledge of the interconnection strengths.

Remark 2: A projection algorithm must be used to ensure that $\beta_k(t) + \hat{\beta}(X_i)$ is bounded away from zero [11]. In addition, projection algorithms may be used to restrict the other parameter estimates. This may be particularly important with c_i so that the magnitude of the switching action does not become undesirably large. In addition, if any of the ideal feedback gains are known, then these gains may be used without adaptive estimation of these gains.

Remark 3: The above results ensure that an adaptive controller does exist for each subsystem which ensures asymptotic tracking of a reference signal. The weak interconnection assumptions often associated with decentralized control system designs based on M-matrix techniques are thus avoided.

Remark 4: To smooth the control action, the discontinuous $\text{sgn}(w_i)$ function may be approximated by a continuous function so that $\nu_i = y_{m_i}^{(r_i)} + k_{i,r_i-1} e_{o_i}^{(r_i-1)} + \cdots + k_{i,0} e_{o_i} + c_i \,\text{sat}(w_i(t)/\epsilon_i) + \eta_i w_i(t)/2$, where $\text{sat}(x) := 1$ if $x > 1$, x if $|x| \leq 1$, -1 if $x < -1$ in (8). In addition, a dead-zone nonlinearity may be incorporated into the estimation of c_i and η_i, so that $\dot{c}_i = \frac{1}{q_{c_i}} |\bar{w}_i|$ and $\dot{\eta}_i = \frac{1}{2q_{\eta_i}} \bar{w}_i^2$, where $\bar{w}_i := w_i - \epsilon_i \,\text{sat}(w_i/\epsilon_i)$ [15]. Using this smoothing algorithm may help reduce the magnitude of the control action; however, asymptotic convergence of the tracking errors will no longer be guaranteed.

IV. Example: An Automated Highway System

Due to increasing traffic congestion, there has been an renewed interest in the development of an AHS in which high traffic flow rates may be safely achieved. Since many of today's automobile accidents are caused by human error, automating the driving process may actually increase the safety of the highway. Vehicles will be driven automatically with onboard lateral and longitudinal controllers. The lateral controllers will be used to steer the vehicles around corners, make lane changes, and perform additional steering tasks. The longitudinal controllers will be used to maintain a steady velocity if a vehicle is traveling alone (conventional cruise control), follow a lead vehicle at a safe distance (car following, see Fig. 2), or perform other speed/tracking tasks. Here we consider the car-following problem in which only tracking information is available (as opposed to information about lead and other subsequent vehicles) to each following vehicle [16]. For more details on intelligent vehicle highway systems (IVHS), see [17] and [18].

TABLE I
Automobile Variables and Parameters

x	vehicle position
v	vehicle velocity
f	applied force in longitudinal direction
$m = 1300kg$	mass of the vehicle
$A_\rho = 0.3Ns^2/m^2$	aerodynamic drag
$d = 100N$	constant frictional force
$\tau = 0.2s$	engine/brake time constant

The dynamics of the car-following system for the ith vehicle may be described by the state vector $X_i = [\psi_i, v_i, f_i]^T$, where $\psi_i = x_i - x_{i-1}$ is the intervehicle spacing between the ith and $i-1$st vehicles, v_i is the ith vehicle's velocity, and f_i is the driving/braking force applied to the longitudinal dynamics of the ith vehicle. The longitudinal dynamics may be expressed as

$$\dot{\psi} = v - v_{i-1} \tag{28}$$

$$\dot{v} = \frac{1}{m}(-A_\rho v^2 - d + f) \tag{29}$$

$$\dot{f} = \frac{1}{\tau}(-f + u_p) \tag{30}$$

where u_p is the control input (if $u_p > 0$, then it represents a throttle input, and if $u_p < 0$, it represents a brake input), and the vehicle variables and parameters are summarized in Table 1 (we assume that the variables and parameters are associated with the ith vehicle, unless subscripts indicate otherwise).

The plant output is $y_p = \psi + L + \rho v$, $L, \rho > 0$. This measurement allows for a velocity-dependent intervehicle spacing [19] due to the ρv term plus an additional constant intervehicle spacing of L. As the velocity of the ith vehicle increases, the distance between the ith and $i-1$st vehicles should increase. A standard good driving rule for humans is to allow an intervehicle spacing of one vehicle length per 10 mph (this roughly corresponds to $\rho = 0.9$). With $\rho \neq 0$, the plant is of relative degree two since

$$y_p^{(2)} = \dot{v}_i + \rho \ddot{v}_i - \dot{v}_{i-1} \tag{31}$$

$$= \frac{1}{m}[-A_\rho v^2 - d + f] + \frac{\rho}{m}\left[-2A_\rho v\dot{v} - \frac{1}{\tau} f\right] + \frac{\rho}{m\tau} u_p - \dot{v}_{i-1}. \tag{32}$$

This is of the form required by the decentralized adaptive controller with

$$\alpha(X) = \frac{1}{m}[-A_\rho v^2 - d + f] + \frac{\rho}{m}\left[-2A_\rho v\dot{v} - \frac{1}{\tau} f\right] \tag{33}$$

$$\beta(X) = \frac{\rho}{m\tau} \tag{34}$$

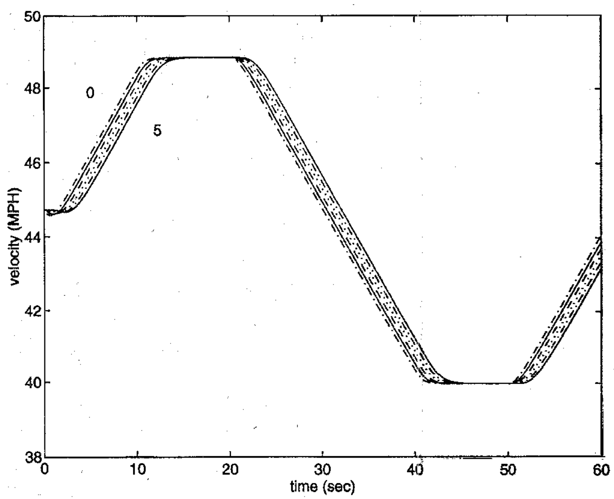

Fig. 3. Velocity profiles of a six-car string of vehicles.

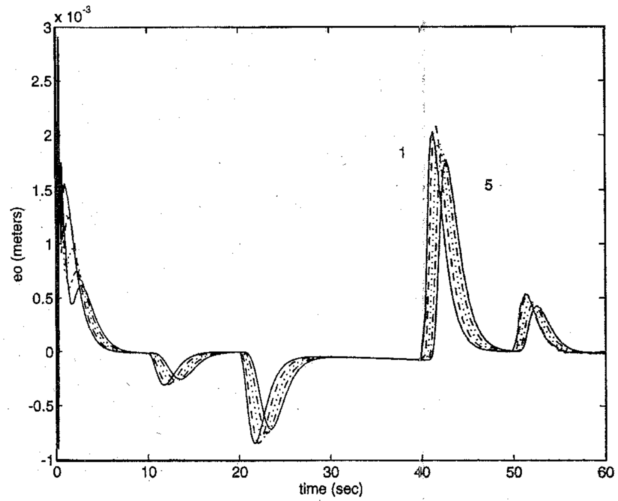

Fig. 4. Spacing errors for the five following cars.

where $\Delta = -\dot{v}_{i-1}$. Since we desire that $y_p \to 0$, here we simply select $y_m := 0$ so that

$$e_o = -y_p. \tag{35}$$

Notice that if $v > 0$ and y_p is forced to zero, then ψ will be negative. This implies that $x_{i-1} > x_i$ (i.e., the $i-1$st vehicle will be ahead of the ith vehicle). Since $\Delta = \frac{1}{\rho}\dot{e}_{o_{i-1}} - \frac{1}{\rho}(v_{i-1} - v_{i-2})$, the interconnections may be bounded with $\gamma_{i,i-1} = 1/\rho$, and $|d_i(t)| \leq |v_{i-1} - v_{i-2}|/\rho$. Also $|v_{i-1} - v_{i-2}|$ is bounded for the ith vehicle since if $i = 0$ (corresponding to the lead vehicle), then we may consider $d_1 = 0$. Thus the above adaptive technique may be applied to the first following car, $i = 1$, with bounded tracking so that $|v_1 - v_0|$ is bounded. Thus this technique may be applied to the second following vehicle, and so on.

For this example, we choose $R_i = \text{diag}\{1, 1\}$ and $L_i(s) = s^2 + 2s + 1$. With $\lambda_i = 1$ and each $\kappa_i = 1$ (tracking performance for each car is weighted equally), using the arguments for the existence of some η large enough for feedback stabilization, we see that $\eta = 1/\rho^2$ is sufficient for diagonal dominance (we shall not adapt η_i since a stabilizing gain is known). From (33), we choose $\zeta_{\alpha_i} = [1, v_i^2, v_i\dot{v}_i, f_i]$ and $\zeta_{\beta_i} = [1]$. The adaptation rates were chosen as $Q_{\alpha_i}^{-1} = \text{diag}\{0.01, 0.01, 0.01, 0.01\}, Q_{\beta_i}^{-1} = [0.01]$, and $q_{c_i}^{-1} = [0.1]$. The parameter estimates were initially set to zero, except for $B_{\beta_i}(0) = 0.01$. In addition, a projection algorithm was used to ensure that $A_{\beta_i} \geq 0.001$, and the smoothing technique of Remark 4 was used with $\epsilon_i = 0.001$.

A string of vehicles with five following vehicles was considered in the simulation analysis with $L = 2$ and $\rho = 0.4$. The vehicles states were initialized with no intervehicle spacing or velocity errors. The velocity profiles for the string of vehicles is shown in Fig. 3 [plots are labeled with lead vehicle (-.-.-), car #1 (—), car #2 (- - -), car #3 (···), car #4 (-.-.-), car #5 (—)]. The intervehicle spacing errors are shown in Fig. 4. The adaptive controllers are able to quickly provide good tracking.

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